

THIRTY-THIRD ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I 1989

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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1. When simplified, the expression $(x + y)^3 - (x - y)^3$ becomes
- a. $2y^3$
 - b. 0
 - c. $6x^2y + 2y^3$
 - d. $12xy + 2y^3$
 - e. $2x^3y^3$
2. Which of the following statements is not true for all values of x ?
- a. $|x| \geq 0$
 - b. $x \leq |x|$
 - c. $|-x| = x$
 - d. $|x| = |-x|$
 - e. $-|-x| \leq 0$
3. If $5x - 3y = 11$, then y equals which of the following expressions?
- a. $(5x - 11)/3$
 - b. $(11 - 5x)/3$
 - c. $5x - 11$
 - d. $(16x)/(-3)$
 - e. $(16x)/3$
4. Which of the following is a factorization of the polynomial $12x^2 + x - 6$?
- a. $6(x - 1)(2x + 1)$
 - b. $6(x + 1)(2x - 1)$
 - c. $(3x + 2)(4x - 3)$
 - d. $3(x - 2)(4x + 1)$
 - e. $(3x - 2)(4x + 3)$
5. Which of the following values of k would guarantee that the equation $2x^2 - kx + 3 = 0$ will have rational roots?
- a. 0
 - b. 5
 - c. 6
 - d. 1
 - e. 2
6. Which of the following is a factorization of the polynomial $8x^3 - y^3$?
- a. $(2x - y)^3$
 - b. $(2x - y)(4x^2 - 2xy + y^2)$
 - c. $(2x - y)(4x^2 + xy + y^2)$
 - d. $(2x - y)(4x^2 + 2xy + y^2)$
 - e. $(2x - y)(2x^2 + 2xy + y^2)$

7. The expression $\frac{2}{\sqrt{5} - \sqrt{7}}$ simplifies to:

- a. $-\sqrt{5} - \sqrt{7}$ d. $\sqrt{7} + \sqrt{5}$
 b. $2(\sqrt{5} - \sqrt{7})$ e. $\sqrt{7} - \sqrt{5}$
 c. $1/(\sqrt{3})$

8. Simplify the following complex fraction.

$$\frac{\frac{1}{p} - \frac{1}{q}}{\frac{1}{p^2} - \frac{1}{q^2}}$$

- a. $\frac{q+p}{pq}$ d. $1/(p+q)$
 b. $p+q$ e. $pq/(q-p)$
 c. $\frac{pq}{p+q}$

9. Simplify the expression:

$$\frac{(x^2)^4 (y^3)^{(2/3)}}{x^{-3} (y^{-1})^{-3}}$$

- a. $\frac{x^6 y^{(11/3)}}{x^{-3} y^{-4}}$ d. $\frac{y}{x^5}$
 b. $\frac{x^{11}}{y}$ e. $\frac{y}{x^{11}}$
 c. $x^{11} y$

10. The solution (x,y) for the system of equations $4(x-1) + 2(y+2) = 0$ is
 $2x = 7 - y$

- a. (0,0) d. (5/3, 1/4)
 b. (-1,0) e. There is no solution.
 c. (3,-2)

11. If y varies directly as x and inversely as the square of z , and y is 4 when x is 2 and z is 16, then find y when x is $1/4$ and z is 2.
- a. $\sqrt{2}$ d. $1/2$
 b. 4 e. 32
 c. $\frac{2\sqrt{2}}{3}$
12. Find the solution to: $\sqrt[3]{y-1} = \sqrt{5}$
- a. $1 + 5\sqrt{5}$ d. $5^{2/3} + 1$
 b. $-1 \pm 5\sqrt{5}$ e. 126
 c. 6
13. Five times the sum of 3 and twice some number, m , can be expressed as:
- a. $15 + 2m$ d. $25m$
 b. $17m$ e. $3 + 10m$
 c. $15 + 10m$
14. Which of the following conditions will make $2x - y$ a negative number?
- a. $x < 0$ d. $2x > -y$
 b. $y > 0$ e. $y > 2x$
 c. $2x > y$
15. What is the minimum y value for the parabola $y = x^2 + 2x - 3$?
- a. -1 d. 0
 b. -4 e. -3
 c. -6
16. The solution for $|3 - 2x| < 4$ is:
- a. $x < -1/2$ d. $-7/2 < x < 1/2$
 b. $x > -1/2$ e. $-1/2 < x < 7/2$
 c. $x < 1/2$

17. Which of the following quadratic equations has complex roots?

a. $2x^2 - 5x + 3 = 0$

d. $2x^2 - x + 3 = 0$

b. $2x^2 + x - 3 = 0$

e. $x^2 - 1 = 0$

c. $2x^2 - x - 3 = 0$

18. Solving for Z in the equation $Z_t Z_p = \frac{Z_o Z}{Z_o + Z}$ will give Z =

a. $Z_t Z_p - (Z_o + 1)$

d. $\frac{Z_t Z_o Z_p}{Z_o - Z_t Z_p}$

b. $Z_t Z_p (Z_o + 1)$

c. $Z Z_{top}$

e. $\frac{Z_t (Z_o - 1)}{Z_p}$

19. What is the range of the function $y = x/|x|$?

a. { 1 }

d. { y | y < 0 }

b. { 1 , -1 }

e. { y | y is not 0 }

c. { y | y > 0 }

20. Which of the following relations is also a function y=f(x)?

a. $y = x^2 + x$

d. $x = y^2$

b. $x = y^2 + y$

e. $x = 3$

c. $x^2 + y^2 = 4$

21. Find the slope of the line $2x - 3y - 18 = 0$.

a. 3/2

d. 2/3

b. - 3/2

e. - 2/3

c. 2

22. Solve:

$$| 4x/5 - 1 | > 3$$

a. $-5/2 < x < 5$

d. $x > 5/2$

b. $x > 5$ or $x < -5/2$

e. $x < -5$ or $x > 5/2$

c. $x > 5$

23. If the length of a side of a square is increased by 4 cm, the perimeter of the new square is 60 cm more than twice the length of a side of the original square. Find the length of a side of the original square.

- a. 5.5
b. 6.5
c. 8
d. 20
e. 22

24. Simplify the following expression:

$$\sqrt[3]{-81x^3} - 3x\sqrt[3]{3} + 5x\sqrt[3]{24}$$

- a. $10x\sqrt[3]{3}$
b. $-10x\sqrt[3]{3}$
c. $-6x\sqrt[3]{3}$
d. $-4x\sqrt[3]{3}$
e. $4x\sqrt[3]{3}$

25. Find the product.

$$\sqrt[4]{x^3y} \cdot \sqrt[4]{xy^2}$$

- a. $\sqrt[8]{x^4y^3}$
b. $\sqrt[16]{x^4y^3}$
c. $x^2y\sqrt{y}$
d. $|x|\sqrt[4]{y^3}$
e. $x^2\sqrt[4]{y^3}$

26. The solution set for the equation $\frac{1}{x} = \frac{8}{x+3} - \frac{2}{x-1}$ is

- a. $\{-3, 1\}$
b. $\{1/5, 3\}$
c. $\{1/3, 5\}$
d. $\{i, -i\}$
e. $\{-3, -1/5\}$

27. Find the product: $(5ax + by)(2ax - 3by)$
- a. $10a^2x^2 + 13abxy - 3b^2y^2$ d. $10a^2x^2 + 3b^2y^2$
 b. $10a^2x^2 - 13abxy - 3b^2y^2$ e. $10ax - 13abxy - 3by$
 c. $10a^2x^2 - 3b^2y^2$
28. Find the quotient obtained by dividing $39 + 8x^3 - 4x$ by $2x + 3$.
- a. $4x^2 + 6x + 7$ d. $4x^2 - 6x + 7$
 b. $4x^2 + 6x - 7$ e. 18
 c. $-4x^2 + 6x - 7$
29. Factor the following polynomial: $3a^2c + 3a^2d^2 + 2b^2c + 2b^2d^2$
- a. $(c + d^2)(3a^2 + 2b^2)$ d. $d^2 + c(2a^2 + 3b^2)$
 b. $(c + d^2)(2a^2 + 3b^2)$ e. $5(a^2c + a^2d^2 + b^2c + b^2d^2)$
 c. $d^2 + c(3a^2 + 2b^2)$
30. Combine into a single fraction in lowest terms.

$$\frac{5x + 2}{x - 6} - \frac{3(x + 4)}{x - 6} - \frac{x - 7}{6 - x}$$

- a. $\frac{x - 3}{x - 6}$ d. $\frac{3x - 17}{6 - x}$
 b. $\frac{x - 3}{6 - x}$ e. $\frac{3x - 17}{x - 6}$
 c. $\frac{7x - 3}{x - 6}$

1. Combine into a single fraction in lowest terms.

$$\frac{3a}{2xy} - \frac{2 - 5x}{3y^3} + 6$$

a.
$$\frac{9ay^2 - 4x - 10x^2 + 36xy^3}{6xy^3}$$

d.
$$\frac{9ay^3 - 4xy + 10x^2y + 36xy^4}{6xy^4}$$

b.
$$\frac{9ay^2 - 4x + 10x^2 + 36xy^3}{6xy^3}$$

e.
$$\frac{3a + 5x + 4}{-xy^2}$$

c.
$$\frac{9ay^3 - 4xy - 10x^2y + 36xy^4}{6xy^4}$$

32. Divide $\frac{2y^2 - 11y + 12}{6y^2 - 6y - 12}$ by $\frac{3y^2 - 14y + 8}{2y^2 - 6y + 4}$.

a.
$$\frac{(2y - 3)(y - 1)}{(y + 1)(3y - 2)}$$

d.
$$\frac{(2y + 3)(y + 1)}{3(y - 1)(3y + 2)}$$

b.
$$\frac{(2y + 3)(y + 1)}{(y - 1)(3y + 2)}$$

e.
$$\frac{(2y - 3)(y - 1)}{3(y + 1)(3y - 2)}$$

c.
$$\frac{(2y - 3)(y - 1)}{6(y + 1)(3y - 2)}$$

33. Solve the following equation.

$$\sqrt{2x^2 - 9} = x$$

a. $x = 3$

d. $x = -3i$

b. $x = 3, -3$

e. $x = 3i, -3i$

c. $x = 3i$

34. Find a quadratic equation whose roots are $3 + 2\sqrt{3}$ and $3 - 2\sqrt{3}$.
- a. $x^2 + 6x + 3 = 0$ d. $x^2 - 6x + 3 = 0$
b. $x^2 - 6x - 3 = 0$ e. $x^2 - 3x - 6 = 0$
c. $x^2 + 6x - 3 = 0$
35. If point A has coordinates (2,3) and point B has coordinates (-5, 6) then the slope m of the line segment joining A and B is:
- a. $3/7$ d. $-7/3$
b. $-3/7$ e. $\sqrt{58}$
c. $7/3$
36. If $f(x) = x^2 - 3x + 2$ then which one of the following statements is false?
- a. The range of f is $\{y \mid y > 0\}$ d. f has no maximum value
b. The graph of f is a parabola. e. The graph of f crosses the x-axis twice.
c. The minimum value of f occurs at $x = 3/2$
37. A ball thrown into the air with a velocity of 32 ft/sec will be above the ground s ft at any time t (measured in seconds) with $s = -16t^2 + 32t$. At what time after it is thrown will the ball hit the ground?
- a. 0.5 sec d. 2.0 sec
b. 1.0 sec e. 3.0 sec
c. 1:25 sec

38. Given the function $f(x) = 2x^5 + 4x^4 - 6x^3 + 2x^2 + 8x - 5$, how many possible positive roots are there?
- a. 3 or 1
b. 4, 2 or 0
c. 5
d. 4
e. 2
39. A company's sales volume (S) per month varies directly as the number (A) of dollars per month spent for advertising, and inversely as the product of the selling price x dollars per unit and the inflation index (I). Express this relationship in an equation.
- a. $S = k(xI)/A$
b. $S = kA/(xI)$
c. $S = kA/(x+I)$
d. $S = kAxI$
e. $S = kAx/I$
40. If $-5 < A < 3$ and $4 < B < 7$, then the maximum value of $|A - B|$ is
- a. 4
b. 12
c. 9
d. 1
e. 7

