

1) Consider a rectangle with vertices  $A$ ,  $B$ ,  $C$ , and  $D$ . The midpoint of the side  $\overline{AB}$  is called  $M$ , and the midpoint of side  $\overline{BC}$  is  $N$ . We join  $A$  with  $N$ , then  $C$  with  $M$ , and we call the intersection of the two segments  $P$ . If the measurement of angle  $\widehat{NAB}$  is  $36^\circ$  and the measurement of angle  $\widehat{CPN}$  is  $35^\circ$ , what is the measurement of angle  $\widehat{MDA}$ ?

- A)  $20^\circ$
- B)  $45^\circ$
- C)  $30^\circ$
- D)  $31^\circ$
- E)  $19^\circ$

2) Given the following sequence of numbers:

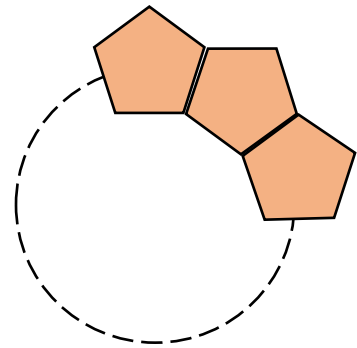
$98, 79, X, 47, 34, 23, Y, 7, 2, Z$

Calculate the value of

$$100X + Y - Z$$

- A) 6213
- B) 6214
- C) 6215
- D) 6414
- E) 6415

3) To decorate a city garden, we want to make a circular path by placing tiles in the shape of regular pentagons next to each other as shown in the figure. If each tile costs 14.99 before tax, and the sales tax in our city is 9.75%, how much will it cost to buy all the tiles needed for the path?



- A) \$131.61
- B) \$148.06
- C) \$149.90
- D) \$164.52
- E) \$180.97

4) In a game of dice, you have to roll two dice at the same time and add the results. One is shaped as a hexahedron, and the other one is a tetrahedron. Each of the dice's sides are numbered using natural numbers, starting at 1. Each turn, if you roll a seven, you get to roll a second time. If you roll a second seven in the same turn, you win. What is the probability of winning in any given turn?



- A) 0
- B)  $\frac{1}{24}$
- C)  $\frac{1}{6}$
- D)  $\frac{1}{49}$
- E)  $\frac{1}{36}$

5) An okapi is a relative of the giraffe. It is hard to weigh an okapi, but easy enough to look at one to measure its height. To estimate the weight of an okapi, multiply its height in inches by 41 and subtract 1980. Express the weight,  $W$ , as a linear function of the height,  $h$ .

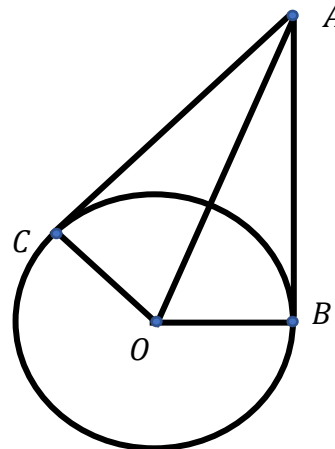
- A)  $W(h) = 41h - 1980$
- B)  $W(h) = 1980h + 41$
- C)  $W(h) = 41(h - 1980)$
- D)  $W(h) = -1980h + 41$
- E) None of the above

6) Lucy Baskets is one of the most promising basketball players in Tennessee with an impressive 83.333333... percent of field throws scored. What is the minimum number of throws a basketball player has to try to be able to obtain the same percentage?

- A) 6
- B) 10
- C) 30
- D) 100
- E) 300

7) In the figure,  $AB$  and  $AC$  are tangential to a circle of radius 5 cm. If  $\overline{AO} = 13$  cm, what is the area of the quadrilateral  $ACOB$ ?

- A)  $30 \text{ cm}^2$
- B)  $32.5 \text{ cm}^2$
- C)  $50 \text{ cm}^2$
- D)  $60 \text{ cm}^2$
- E)  $65 \text{ cm}^2$



8) Find the  $100^{\text{th}}$  number in the sequence: 2, 5, 10, 17, 26, 37, 50, ...

- A) 9,999
- B) 10,000
- C) 10,001
- D) 10,002
- E) 10,003

9) Rahul enters an elevator initially on a certain floor. He goes down 4 floors, then goes up  $\frac{1}{3}$  of his initial floor number. When he finally goes down  $\frac{1}{6}$  of his initial floor number, he finds himself back at the initial floor. In what floor number did Rahul start his journey?

- A) 6
- B) 12
- C) 18
- D) 24
- E) 30

10) What is the simplest form of the following expression?

$$\frac{[(a + b)^2 + (a - b)^2] + 2ab}{a^3 - b^3}$$

A)  $\frac{2}{a-b}$

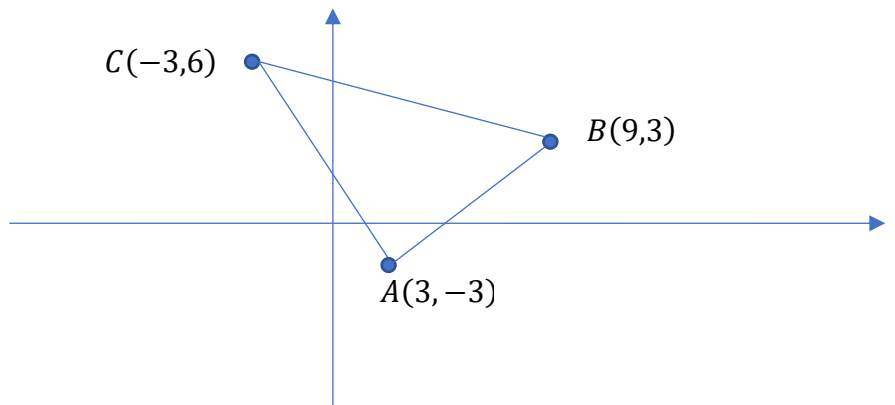
B)  $\frac{2}{a+b}$

C)  $\frac{6ab}{a^3-b^3}$

D)  $\frac{6ab}{a^3+b^3}$

E) 0

11) If the following triangle  $ABC$  is dilated by  $\frac{1}{3}$  about the origin, and then translated to the left by 2 units, the new coordinates of  $A$ ,  $B$ ,  $C$  will be, respectively:



A) (1, 1), (3, 3), (-1, 4)

B) (3, -1), (5, 1), (1, 2)

C) (1, -3), (3, -1), (-1, 0)

D) (-1, -1), (1, 1), (-3, 2)

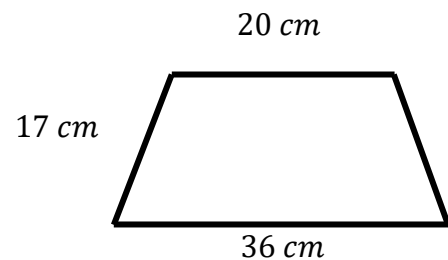
E) (-3, 2), (1, 1), (-1, -1)

12) What is the following expression's simplest form?

$$\sqrt[4]{\sqrt[3]{\sqrt{\frac{a^{72}b^{-48}}{a^{-24}c^{-96}}}}}$$

- A)  $\frac{a^2c^4}{b^2}$
- B)  $\frac{a^4c^4}{b^2}$
- C)  $\frac{a^4c^3}{b^2}$
- D)  $\frac{a^4b^4}{c^2}$
- E)  $\frac{a^4b^2}{c^4}$

13) The base of a prism is an isosceles trapezoid shown below. Find the volume of the prism if its height is 8 cm. (Figure not to scale.)



- A)  $840 \text{ cm}^3$
- B)  $2,720 \text{ cm}^3$
- C)  $3,360 \text{ cm}^3$
- D)  $3,808 \text{ cm}^3$
- E)  $6,720 \text{ cm}^3$

14) One fifth of a class prefers the color blue, a sixth of the class prefers red, and those who prefer green constitute one half of the sum of those who prefer blue and red. The remaining 27 students prefer yellow. How many students are in the class?

- A) 60
- B) 54
- C) 48
- D) 36
- E) 30

15) We divide a circular garden of diameter  $16\text{ m}$  into four sections as in the picture. We want to plant tulips on section number 2; however, we don't know its exact area. What is the best estimation for the area of section 2?



- A)  $16\pi\text{ m}^2$
- B)  $26\pi\text{ m}^2$
- C)  $28\pi\text{ m}^2$
- D)  $32\pi\text{ m}^2$
- E)  $64\pi\text{ m}^2$

16) If we know a bag of tulip bulbs will cover an area of  $6.12\text{ m}^2$ , how many bags should we buy at the store in order to cover  $9\pi\text{ m}^2$ ?

- A) 1
- B) 2
- C) 3
- D) 4
- E) 5

17) At a concert, 55% of the tickets sold for their initial price, and 40% were sold at half the price. There were 20 unsold tickets that were given away for free. A total of \$7,200 were collected. What was the initial price of the tickets?

- A) \$14
- B) \$24
- C) \$75
- D) \$346,99
- E) \$96

18) The number 2020 does not have a natural number square root. What is the lowest natural number we could multiply by 2020 so the product is a perfect square?

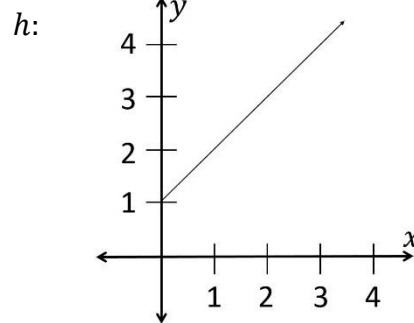
- A) 4
- B) 5
- C) 21
- D) 505
- E) 2020

19) The following functions  $f$ ,  $g$ , and  $h$  represent the distance,  $y$ , over time,  $x$ , travelled by three different objects (measured in the same units):

$f$ :

$x$	$y$
0	1
5	6
10	11

$g: x = \frac{y}{4} - \frac{3}{8}$



Which object is moving faster?

- A) The object represented by function  $f$
- B) The object represented by function  $g$
- C) The object represented by function  $h$
- D) The objects represented by the functions  $f$  and  $h$ , which are traveling at the same speed
- E) The answer cannot be determined with the information provided

20) In a certain tribe, instead of using money, they have a bartering system. Some of the rules are the following:

- A necklace and a shield together can be exchanged for a spear.
- A spear can be exchanged for three knives.
- Two shields can be exchanged for three knives.

How many necklaces is a spear worth in the tribe?

- A) 2
- B) 3
- C) 4
- D) 5
- E) The answer cannot be determined with the information provided

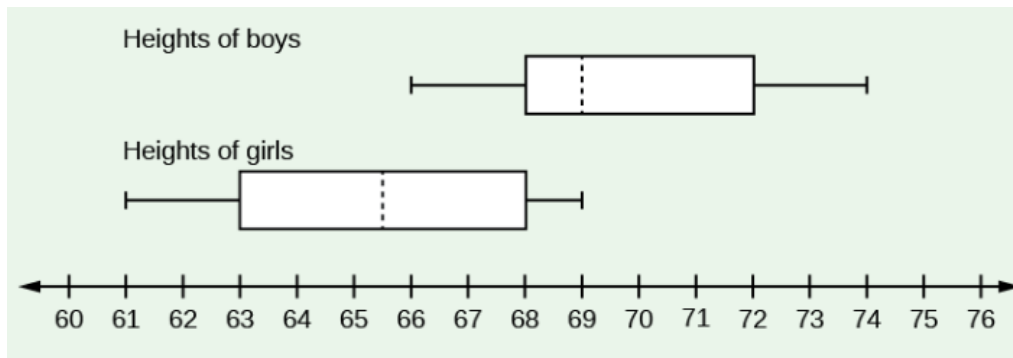
21) What is the sum of the digits of the number  $10^n - 2020$  for  $n \geq 4$ ?

- A)  $9n + 24$
- B)  $9(n + 1) + 24$
- C)  $9(n - 2) + 24$
- D)  $9(n - 3) + 24$
- E)  $9(n - 4) + 24$

22) Suppose a book has  $n$  pages, which are numbered from 1 to  $n$ . If the total number of digits appearing as the page numbers is  $2n$ , what must be true about the number  $n$ ?

- A)  $0 < n < 9$
- B)  $9 < n < 99$
- C)  $n = 99$
- D)  $99 < n < 999$
- E)  $n = 999$

23) The following box plots represent the heights of the boys and girls of a classroom:



With the information provided by the diagram, which of the following statements can we certainly say is true about the sample?

- A) The class consists of 8 girls and 8 boys
- B) The shortest boy is taller than the tallest girl
- C) The tallest girl is as tall as the mean boy
- D) The range of the heights in the class is one foot and one inch
- E) We cannot determine any of the above statements as true



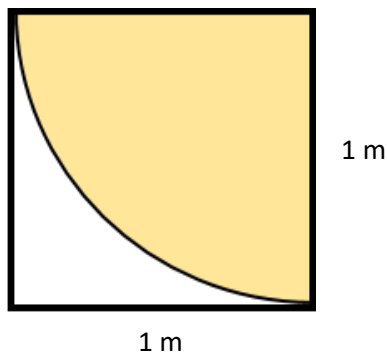
24) What is true about the number of faces (F), vertices (V), and edges (E) in a pentagonal prism?

- A)  $V - E = 2 - F$
- B)  $V + E - F = 2$
- C)  $V = 15, F = 7, E = 10$
- D)  $V + E + F = 2$
- E) None of the above

25) Consider the function  $f(x) = \diamond x^2 + \nabla x + \circ$  where  $\diamond$ ,  $\nabla$ , and  $\circ$  are integers. Which of the following statements is true about the function?

- A)  $f(x)$  is a linear function
- B)  $f(x)$  is a quadratic function
- C)  $f(x)$  is not a function
- D)  $f(x)$  is a cubic function
- E) We do not have enough information to determine the nature of the function.

26) Look at this weird dart board. If we blindfold Ms. Fortune (who never misses the board) and ask her to throw a dart at it, what is the probability that she does not hit the shaded area?



- A)  $\frac{\pi}{4}$
- B)  $\frac{4-\pi}{4}$
- C)  $\frac{4}{\pi} - 1$
- D)  $1 - \frac{\pi}{2}$
- E)  $\frac{1}{3}$

27) A group of students were planning a field trip. Each day, they wanted to share 1 bottle of water among each 4 students for breakfast, one between each 2 for lunch, and another one among each 3 for dinner. Assume that each bottle is fully consumed and shared equally at each meal. What is the minimum number of water bottles they needed per day?

- A) 9
- B) 12
- C) 13
- D) 14
- E) 24

28) Rowena, Helga, and Godric have some Defense against the Dark Arts homework due tonight. If they did it individually, Rowena would finish in 20 minutes, Godric in half an hour, and Helga in an hour. To save some time, they decide to do it together. Assume that they are perfectly efficient in dividing the tasks among them. How long will it take them to do the homework? Do not count the time required to make copies of the work so that each student has a copy.

- A) 8 minutes
- B) 10 minutes
- C) 15 minutes
- D) 20 minutes
- E) 37 minutes

29) Two nomads were resting at an oasis when a third one, Nasrudin, showed up. Nasrudin asked them for some food since he had run out of it during his journey. The first nomad had 5 pieces of bread and the second one had 3. The three nomads shared the bread evenly. What is the ratio of the amount of bread given to Nasrudin by the first nomad to the amount of bread given by the second nomad?

- A) 1 : 7
- B) 7 : 1
- C) 2 : 6
- D) 6 : 2
- E) 1 : 1

30) Consider the difference of two numbers and the sum of the same two numbers. Which of the following statements is true?

- A) It is impossible for the difference of two numbers to be greater than the sum of two numbers.
- B) It is possible for the difference of two numbers to be greater than the sum, but both of the numbers have to be negative for this to be true.
- C) It is possible for the difference of two numbers to be greater than the sum, but the difference must be negative for this to be true.
- D) It is possible for the difference of two numbers to be greater than the sum, but the subtrahend must be negative for this to be true.
- E) It is possible for the difference of two numbers to be greater than the sum, but the minuend must be negative for this to be true.