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# Transport Properties

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# Definitions

As the name implies transport properties involve the transfer of a property from one place to another.

The property is transferred from an area of “higher amount” to an area of “lower amount”. The change from higher amount to lower amount defines what we call a *gradient*.

The rate of migration of the property is measured by its *flux* (J), the quantity of that property passing through unit area per unit time. The flux is proportional to the gradient and the proportionality constant defines a coefficient of the property that is sometimes referred to as the property itself.

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# The 3 major transport properties

**Diffusion** – the migration of matter across a concentration gradient

**Thermal conduction** – the migration of energy across a temperature gradient

**Viscosity** – the migration of momentum across a velocity gradient

**These definitions lead to the following derivative equations in terms of the flux and the gradient.  
(See the lecture notes for equations and units)**

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# Newtonian and non-Newtonian Fluids

## Newton's Law of Viscous Flow

$$F = hA \frac{dv_x}{dz}$$

If  $\eta$  is constant the fluid is called Newtonian. If  $\eta$  is not constant and depends on the gradient then the fluid is non-Newtonian. Water is a Newtonian fluid.

Examples of non-Newtonian fluids are:

- polymer solutions and melts
- slurries of ore, sand, coal
- cement, asphalt
- paint, ketchup, peanut butter
- blood, saliva

## Dilatant and Thixotropic

If  $\eta$  increases as the shear force on the fluid increases then the fluid is called dilatant.

- wet sand, slime

In wet sand it is the particle spacing which determines this result. In slime the molecules are coiled and stress elongates them, thus increasing the viscosity.

If  $\eta$  decreases as the shear force on the fluid increases then the fluid is called thixotropic.

- paint, tomato ketchup, toothpaste, peanut butter

This is the more common behavior because the shear force destroys the weak secondary (van der Waals) forces which hold the molecules together.

## Temperature Dependence

The temperature dependence of  $\eta$  is different for gases and liquids. For liquids as  $T \uparrow$ ,  $\eta \downarrow$ ; For gases as  $T \uparrow$ ,  $\eta \uparrow$ . Let's explore the nature of these trends, starting with liquid viscosity.

### Liquid viscosity

Liquid and gas viscosity are different in that a liquid molecule must escape from its neighbors in order to move. This means that a molecule moves only if it acquires at least a minimum energy. The probability that a molecule has at least the necessary energy  $E_a$

to move is proportional to  $e^{\frac{-E_a}{RT}}$ . The mobility of a molecule should follow this type of temperature dependence.

It is also true that the coefficient of viscosity ( $\eta$ ) is inversely proportional to the mobility.

Thus  $\eta$  is proportional to  $e^{\frac{E_a}{RT}}$  and  $\eta = A e^{\frac{E_a}{RT}}$ . The equation shows us that temperature and  $\eta$  are inversely proportional.

If we take the natural log of both sides then

$$\ln h = \ln A + \frac{E_a}{RT}$$

A graph of  $\ln \eta$  vs  $1/T$  gives  $E_a/R$ .

### **Measuring liquid viscosity**

A common type of device to measure liquid viscosity is called the Ostwald viscometer, in which a liquid is allowed to flow through a small bore capillary between 2 etched lines and the time is measured.

From Poiseuille's equation for  $\eta$  we see that  $\eta$  is directly proportional to time ( $t$ ) and the pressure ( $\Delta P$ ). The pressure difference is proportional to the density ( $D$ ) of the liquid being measured. The other parameters in Poiseuille's equation can be considered to be a constant using the Ostwald viscometer so the final equation reduces to

$$\eta = k D t$$

where  $k$  is the proportionality constant. Once a reference sample of known viscosity is analyzed, density and time readings for other samples gives the viscosity.

$$\frac{n}{n_r} = \frac{Dt}{D_r t_r}$$

## **Solution viscosity**

If 2 liquids are mixed the behavior may be classified as “ideal” or “non-ideal”. If the interactions between the liquid particles are about the same as the interactions for each liquid, the viscosity of the mixture will be between the viscosities of the pure liquids.

If the 2 liquids have greatly different particle interactions, the viscosity of the mixture may be greater or less than either of the 2 liquids.

## Gas viscosity

We will derive in class the following result for gas viscosity  $\eta = \frac{1}{3} \rho \langle c \rangle \lambda$  which can be simplified to

$$\eta = \frac{2}{3} \left( \frac{mkT}{p\sigma} \right)^{1/2}$$

$m$  is the mass per particle (kg)

$k$  is the Boltzmann constant (J/K)

$T$  is the absolute temperature in Kelvin

$\sigma$  is the collision cross section ( $\text{m}^2$ ) roughly equaling  $\pi (\text{diameter})^2$

From this equation we see the following:

- $\eta$  depends on the mass of the particle, the molecular size, and the temperature
- It appears that density or pressure does not affect the viscosity. At higher densities more molecules jump from one layer to the next, but the mean free path is smaller and each jump involves the transport of less momentum. These two effects cancel each other out.
- $\eta$  increases with the square root of temperature, which is the opposite of liquid viscosity. Since the molecules don't have to overcome the attraction of nearest neighbors, the temperature dependence is in the average speed which has a square root dependence.
- Finally if mass is held constant, the larger molecule (larger  $d$ ) would have the lower viscosity. An increase in  $d$  reduces the mean free path and thus reduces the transfer of momentum.