

# 1999 EIGHTH GRADE MATHEMATICS COMPETITION

AUSTIN PEAY STATE UNIVERSITY  
CLARKSVILLE, TENNESSEE

MIDDLE TENNESSEE STATE UNIVERSITY  
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UNIVERSITY OF TENNESSEE AT MARTIN  
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Scoring Formula:  $4R - W + 40$

## DIRECTIONS:

For each problem there are 5 possible answers listed. You are to work the problems, determine the correct answer, and indicate your choice on the separate answer sheet provided.

## SAMPLE:

1. If  $x + 1 = 2$ , then  $x$  equals

- a) 0
- b) 2
- c) -1
- d) 1
- e) none of the above

	A	B	C	D	E
1	①	②	③	●	⑤
	A	B	C	D	E
2	①	②	③	④	⑤
	A	B	C	D	E
3	①	②	③	④	⑤

The correct answer is 1, which is d); so you would answer this problem by darkening the space on the answer sheet corresponding with this choice.

If you change your mind about your answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin. When you have finished one page, go on to the next. The working time for the entire test is 60 minutes.

1.  $1 + 2 + 3 + 4 + 5 = 15$ ;  $2 + 3 + 4 + 5 + 6 = 20$ ;  $3 + 4 + 5 + 6 + 7 = 25$ . The sum of any 5 consecutive Natural Numbers is a multiple of 5. Which of the following numbers is the least of the five consecutive Natural Numbers whose sum is 1335?

- (a) 265
- (b) 267
- (c) 269
- (d) 135
- (e) 1035

2. Fifteen percent of the students in the sixth period gym class wear glasses. There are 34 students in the sixth period gym class who do not wear glasses. How many students are in the sixth period gym class?

- (a) 28.9
- (b) 39.1
- (c) 40
- (d) 49
- (e) 85

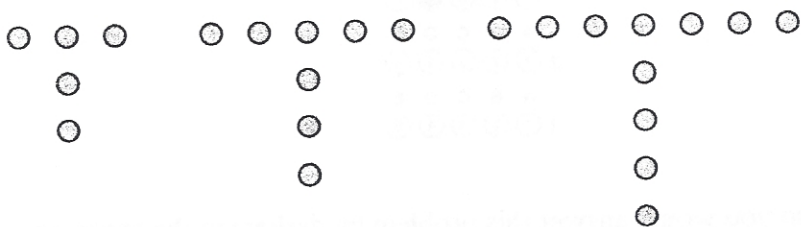
3. How many of the first 100 counting numbers are neither multiples of 4 nor multiples of five?

- (a) 40
- (b) 45
- (c) 55
- (d) 60
- (e) 95

4. Of the five numbers given in the choices, which is the least?

- (a)  $\frac{2}{3}$
- (b)  $\left(\frac{2}{3}\right)^2$
- (c)  $\frac{5}{6}$
- (d)  $\left(\frac{5}{6}\right)^2$
- (e)  $\frac{19}{20}$

5. The first three T-numbers are shown here.



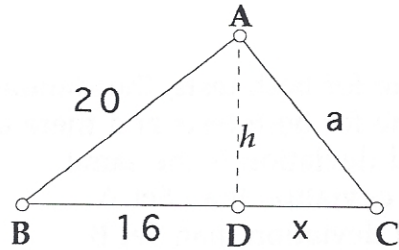
There are 5 dots in the first T-number; 8 dots in the second T-number; and 11 dots in the third T-number. How many dots are in the 100th T-number?

- (a) 111
- (b) 108
- (c) 300
- (d) 302
- (e) 303

6. What is the least prime factor of  $3187 + 5775$ ?

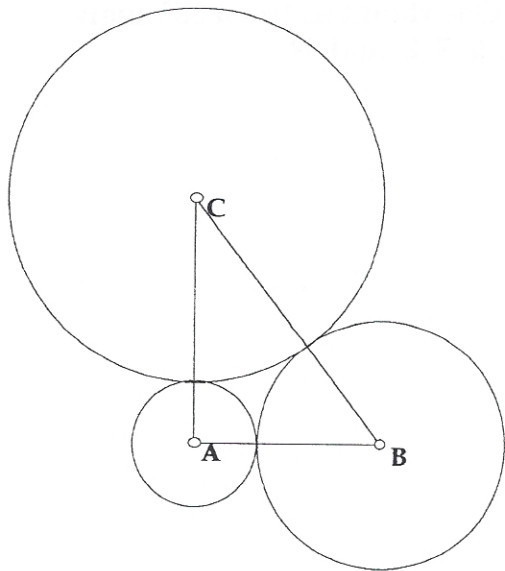
- (a) 2
- (b) 3
- (c) 5
- (d) 7
- (e) 13

7. Triangle ABC has a right angle at vertex A. The altitude,  $h$ , to the hypotenuse of Triangle ABC separates it into two smaller right triangles that are similar to the Triangle ABC.  $BD = 16$ . The length of one leg of the largest triangle, AB, is 20. What is the length of the other leg,  $a$ ?



- (a) 9                      (b) 12                      (c) 15                      (d) 16                      (e) 20

8. Triangle ABC has its vertices at the centers of three circles that are tangent to each other. If  $AB = 6$ ,  $CA = 8$ , and  $BC = 10$ , what is the radius of the smallest circle?



- (a) 1                      (b) 2                      (c) 2.5                      (d) 3                      (e) 3.5

9. If  $ab = 12$  and  $a^2 + b^2 = 25$ , then what is the value of  $(a + b)^2$ ?

- (a) 49                      (b) 25                      (c) 37                      (d) 24                      (e) 144

10.  $3^1 = 3$ ;                       $3^2 = 9$ ;                       $3^3 = 27$ . What digit is in the ones position of  $3^{1999}$ ?

- (a) 1                      (b) 3                      (c) 5                      (d) 7                      (e) 9

11. The scores for two sets of tests are given here:

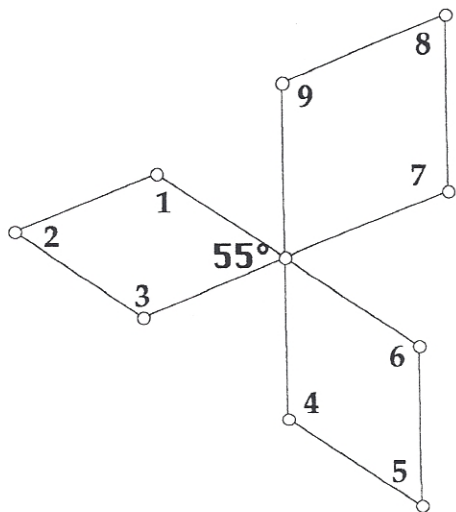
Set A: 75, 75, 75, 80, 80, 80, 80, 85, 85, 85, 85, 90, 90, 90, 90

Set B: All 15 students made 83 on the test.

Which statement is true?

- (a) Because the mean is the same for both tests, the standard deviation is the same.
- (b) Because the mean is the same for both tests and there are the same number of tests in each group, the standard deviation is the same.
- (c) Set B has a greater standard deviation than Set A.
- (d) Set A has a greater standard deviation than Set B.
- (e) It is impossible to compare the standard deviations without calculating it for each set of scores.

12. Three rhombuses have a vertex in common as shown. In the figure, each pair of segments that appears to be collinear is collinear. One rhombus has a  $55^\circ$  angle. What is the sum of the measures of angles 1, 2, 3, 4, 5, 6, 7, 8, and 9?



- (a)  $360^\circ$
- (b)  $540^\circ$
- (c)  $720^\circ$
- (d)  $900^\circ$
- (e)  $1080^\circ$

13. A manufacturer made 47 ounces of perfume. The company puts  $\frac{3}{8}$  ounce of perfume in each bottle. After filling as many bottles as possible, how many ounces of perfume will be left over?

- (a)  $\frac{1}{8}$  ounce
- (b)  $\frac{1}{4}$  ounce
- (c)  $\frac{1}{3}$  ounce
- (d)  $\frac{1}{5}$  ounce
- (e)  $\frac{1}{6}$  ounce

14. A hole is punched in a piece of paper. Then the paper is folded in half and another hole is punched in a different place causing two new holes in the piece of paper. The total number of holes is now three. Using information in the table, what is the TOTAL number of holes when the number of folds is 8?

Number of folds	0	1	2	3
Number of new holes	1	2	4	8
Total number of holes	1	3	7	15

- (a) 512      (b) 511      (c) 256      (d) 255      (e) 128

15. If the figure shown below represents  $\frac{6}{5}$ , which of the figures shown below would represent one half?



- (a)      (b)      (c)      (d)      (e)

16. There are twelve \$1-bills, nine \$5-bills, two \$10-bills, and one \$20-bill mixed up in a paper bag. You may reach in and choose two of them at random. (You will draw both out without replacing either.) What is the probability you get less than \$10?

- (a)  $\frac{1}{12}$       (b)  $\frac{21}{24}$       (c)  $\frac{11}{46}$       (d)  $\frac{29}{46}$       (e)  $\frac{1}{24}$

17. Mr. Bradley rotates the tires on his truck regularly. When the truck was new, it had five tires; one was used as a spare. He rotated the tires every 4000 miles. (The left front went to the right front; the right front went to the right back; the right back went to the bed to be the spare; the spare went to the left back; and the left back went to the left front.) How many miles of wear did each tire have after Mr. Bradley had driven the car 40,000 miles?

- (a) 4000 miles      (b) 8000 miles      (c) 10,000 miles      (d) 20,000 miles      (e) 32,000 miles

18. Jay paid \$36 for a coat. He had an employee discount that allowed him to take 40% off the sale price. The sale price was 25% off the original price. What was the original price of the coat?

- (a) \$85                      (b) \$80                      (c) \$75                      (d) \$70                      (e) \$63

19. A ball and a book cost \$3.50. A book and a basket cost \$6.50. A basket and a ball cost \$6.00. How much does the ball cost?

- (a) \$1.00                      (b) \$1.50                      (c) \$2.00                      (d) \$2.50                      (e) \$3.00

20. Tara was fouled at the buzzer of the basketball championship game with her team behind by 1 point. She was awarded two free throws, which are worth one point each if she makes them. Tara has made 80% of her free throws this season. Based upon that percentage, what is the probability that her team will win in regulation time?

- (a) 0.16                      (b) 0.20                      (c) 0.36                      (d) 0.64                      (e) 0.8

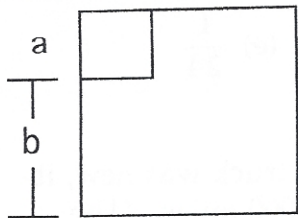
21. Calculate  $(-3 + 7)(2 - 6)^2$ .

- (a) 64                      (b) -64                      (c) 256                      (d) -256                      (e) -32

22. A human being breathes approximately six quarts of air every minute. 14,625,000 people live in New York City. About how many gallons of air do New Yorkers breathe per second?

- (a) 40,625 gal.                      (b) 243,750 gal.                      (c) 365,625 gal.                      (d) 812,500 gal.                      (e) 9,750,000 gal.

23. If the area of the smaller square is  $\frac{1}{9}$  of the area of the larger square, what is the ratio of a to b?



- (a) 1 to 9                      (b) 1 to 8                      (c) 1 to 4                      (d) 1 to 3                      (e) 1 to 2

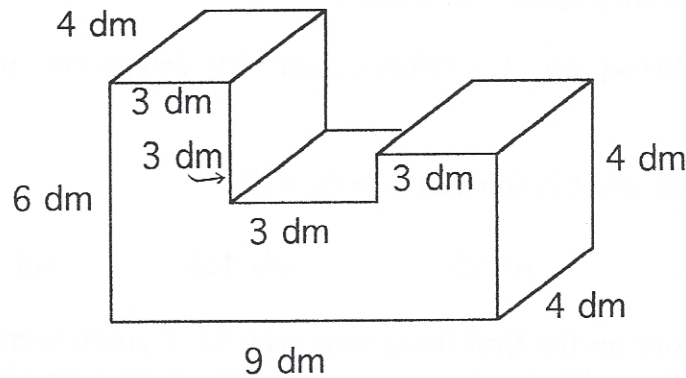
24. It takes Chad 1 hour to mow the lawn. It takes Curt 45 minutes to mow half the lawn. How long would it take both of them working together to mow the lawn?

- (a) 52 minutes                      (b) 1 hour                      (c) 45 minutes                      (d) 36 minutes                      (e) 30 minutes

25. Adult tickets for a play cost \$5. Student tickets for the play cost \$4. The ratio of adult tickets to student tickets sold was 1:2. If the value of all the tickets sold was \$832, how many tickets were sold altogether?

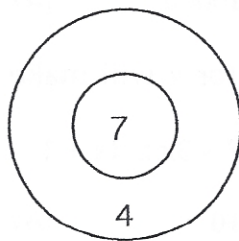
- (a) 61                      (b) 93                      (c) 192                      (d) 279                      (e) 366

26. Find the volume of the solid shown here. (All angles are right angles. There are no gaps you cannot see.)



- (a) 93,312 cubic decimeters  
 (b) 216 cubic decimeters  
 (c) 156 cubic decimeters  
 (d) 144 cubic decimeters  
 (e) 39 cubic decimeters

27. The score on this dartboard is the sum of the points made by each dart. What is the highest score below 100 that is IMPOSSIBLE to score on the given dart board? (Assume you may throw as many darts as you would like.)



- (a) 26                      (b) 23                      (c) 22                      (d) 17                      (e) 12

28. Which of the following types of triangles is impossible?

- (a) right isosceles  
 (b) acute scalene  
 (c) obtuse equilateral  
 (d) right scalene  
 (e) obtuse isosceles

29. At 225 miles per hour, how long would it take a race car to complete a  $2\frac{1}{4}$  mile lap at the race track?

- (a) 36 seconds (b) 37 seconds (c) 40 seconds (d) 42 seconds (e) 50 seconds

30. A cylindrical tube is going to be made from a rectangular piece of heavy paper. The tube should be 20 cm tall and 8 cm in diameter. What is the area of the rectangle needed? (You may ignore overlaps.)

- (a)  $160\pi$  sq. cm (b)  $320\pi$  sq. cm (c)  $1280\pi$  sq. cm (d) 160 sq. cm (e)  $80\pi$  sq. cm

31. If  $\sqrt{x+5} = 5$ , then what is the value of  $(x+5)^2$ ?

- (a)  $\sqrt{5}$  (b) 5 (c) 25 (d) 125 (e) 625

32. Lynn's average score on the first three tests was 92. Lynn's homework score is 96. There is one more test. The final grade is determined by averaging the five grades--4 tests and the homework. If the final grade is at least 94, the student gets an A in the course. What is the least score Lynn needs on the last test to make an A in the course?

- (a) Lynn can't make an A. (b) 100 (c) 98 (d) 96 (e) 94

33. In base two, thirteen is written 1101, fourteen is written 1110, fifteen is written 1111. How would our base-ten number twenty be written in base two?

- (a) 10100 (b) 11000 (c) 10000 (d) 11100 (e) 11110

34. Which one of the given values for  $x$  will make the inequality true?

$$17 - 3x \geq 4x + 1$$

- (a) 20 (b) 15 (c) 10 (d) 5 (e) 0

35. Which of the following numbers is prime?

- (a) 111 (b) 112 (c) 113 (d) 711 (e) 1005

36. Find the least possible counting number,  $N$ , such that  $180 \cdot N$  is a perfect cube.

- (a) 50 (b) 75 (c) 150 (d) 180 (e) 196



37. Keys of different shapes are designed by choosing from several patterns for each of their parts. The keys of General Motors cars have 6 parts. Each part can have one of three possible shapes. How many different key designs are possible?

- (a)  $6 \times 3$       (b)  $6 + 3$       (c)  $6^3$       (d)  $3^6$       (e) 6

38. Which set of numbers could not be the lengths of three sides of a triangle?

- (a) 4, 4, 4      (b) 3, 3, 2      (c) 2, 3, 4      (d) 1, 2, 3      (e) 5, 6, 7

39. The plane can be covered (tessellated) with congruent squares. Which of the following statements is false?

- (a) The plane can be covered (tessellated) with congruent scalene triangles.  
 (b) The plane can be covered (tessellated) with congruent regular pentagons.  
 (c) The plane can be covered (tessellated) with congruent isosceles triangles.  
 (d) The plane can be covered (tessellated) with congruent regular hexagons.  
 (e) The plane can be covered (tessellated) with congruent trapezoids.

40. Peter lives at the bottom of a steep hill. He gets on his bicycle and pedals up the hill. As he pedals up the hill, he gets tired and slows down, but he does NOT stop. When he finally gets to the top of the hill, he coasts down the other side. Which graph illustrates his speed as time passes from the time he got on his bicycle until he reached the bottom of the far side of the hill?

