2018 8ᵗʰ Grade Math Contest

1. What is the 100ᵗʰ number in this sequence?   3, 7, 13, 21, 31, 43, 57...?
   A. 4,951
   B. 9,901
   C. 10,101
   D. 11,201
   E. 247,801

2. Simplify the following expression:
   $$\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-a}}}$$
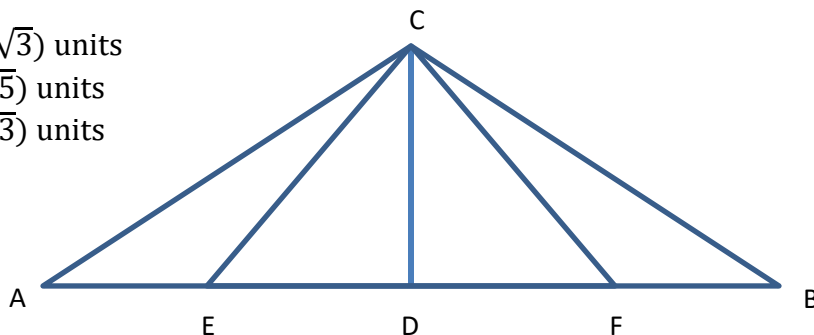
   A. $a - 1$
   B. $1 - a$
   C. a
   D. $\dfrac{1}{1-a}$
   E. $\dfrac{1}{a-1}$

3. At 10:00 A.M. Milo started walking due north from his house. He stopped to rest a few times, and in fact, one of the times he rested was from 10:55 A.M. – 11:0 A.M.0. At 10:55 A.M., he was exactly "a" miles north of his house. At 11:00 A.M. however, he started walking northward again and walked at a constant pace for 3 hours. If y is the number of miles Milo is north of his house at time t, where t is measured in hours since 11:00 A.M., the equation relating y and t is:
   $y = 5t + 4$ for $0 \le t \le 3$

   What is the value of "a" and what is Milo's average speed during the entire trip from 10:00 A.M. until 2:00 P.M.?

   A.   a = 4, average speed during entire trip is 4.75 mi/hr
   B.   a = 4, average speed during entire trip is 5.75 mi/hr
   C.   a = 5, average speed during entire trip is 4.75 mi/hr
   D.   a = 5, average speed during entire trip is 5.75 mi/hr.
   E.   a = 3, average speed during entire trip is 4.25 mi/hr.

4. In the figure below, segment AB is perpendicular to segment CD. Segment CD is 2 units long, segment CF is 3 units long and segment CB is 4 units long. How long is segment FB? (The figure is not necessarily drawn to scale.)

   A. $(2\sqrt{5} - 2\sqrt{3})$ units
   B. $(2\sqrt{3} - \sqrt{5})$ units
   C. $(2\sqrt{5} - \sqrt{3})$ units
   D. $2\sqrt{3}$ units
   E. $2\sqrt{5}$ units

5. Consider the fractions $\frac{13}{17}$ and $\frac{1}{2^{20}}$. Select the true statement.
   A. The decimal representation of each of these fractions terminates.
   B. The decimal representation of each of these fractions is non-terminating but repeats a sequence of digits.
   C. The decimal representations of each of these fractions is non-terminating and non-repeating.
   D. The decimal representation of $\frac{13}{17}$ terminates, and the decimal representation of $\frac{1}{2^{20}}$ is nonterminating, but repeats a sequence of digits.
   E. The decimal representation of $\frac{1}{2^{20}}$ terminates, and the decimal representation of $\frac{13}{17}$ is nonterminating, but repeats a sequence of digits.

6. Quantity A and B start out having the same value. Quantity A doubles every 5 minutes. Quantity B triples every 15 minutes. After how many minutes will quantity B exceed quantity A?

   A. After 6 minutes
   B. After 45 minutes
   C. After 90 minutes
   D. It depends on the starting value.
   E. Quantity B will never exceed Quantity A.

7. If quantity A is directly proportional to quantity B, which of the following statements must be true?
   I. When quantity A has a value of 0, quantity B has a value of 0.
   II. When quantity A doubles, quantity B also doubles.
   III. When quantity A increases by 1 unit, quantity B also increases by 1 unit.

   A. I only
   B. I and II only
   C. I and III only
   D. II and III only
   E. I, II, and III must all be true.

8. If $x = \sqrt{2 - \sqrt{2}}$, then x is a solution of which of the following equations?
   A. $x^4 - 2 = 0$
   B. $x^4 + 2 = 0$
   C. $4x^4 - x^2 + 2 = 0$
   D. $x^4 - 4x^2 + 2 = 0$
   E. None of the above.

9. In the figure below, a circle is inscribed inside a regular hexagon. The diameter of the circle is 2 units. What is the area of the hexagon?

    A. 3 square units

    B. $\dfrac{13\pi}{12}$ square units

    C. $2\sqrt{3}$ square units

    D. $3\sqrt{2}$ square units

    E. $4\sqrt{3}$ square units
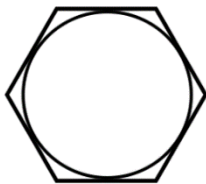
10. There are 2.54 cm in 1 in. How many square cm are in 1 square yard (to the nearest hundredth of a square cm)?

    A. 91.44 square cm

    B. 194.70 square cm

    C. 232.26 square cm

    D. 3,343.68 square cm

    E. 8,361.27 square cm

11. Lian baked some cookies. His sister Ye ate half of them. His friend Arthur ate $\frac{2}{5}$ of what was left. His cousin Anna ate $\frac{1}{3}$ of what was left after Arthur ate his. Then x cookies remained. How many cookies did Lian bake?

    A. $4x$

    B. $5x$

    C. $8x$

    D. $15x$

    E. $30x$

12. A quart of cocoa drink is 70% milk and 30% cocoa powder. Someone pours a pint (half quart) of cocoa drink into the container that has the original quart of cocoa drink. However the pint consisted of 80% milk and 20% cocoa powder. Then someone pours a cup (half-pint) of milk into the resulting mixture. What is the strength of the final mixture (to the nearest percentage)?

    A. 77 % milk, 23% cocoa

    B. 75% milk, 25% cocoa

    C. 80% milk, 20% cocoa

    D. 67% milk, 33 % cocoa

    E. 65% milk, 35% cocoa

13. On Zoo Island a card game is played with the deck of 12 cards below. The names of the cards are 1 of Bears, 2 of Bears, 3 of Bears, 4 of Bears, 1 of Elephants, 2 of Elephants, 3 of Elephants, 4 of Elephants, 1 of Rhinos, 2 of Rhinos, 3 of Rhinos and 4 of Rhinos.
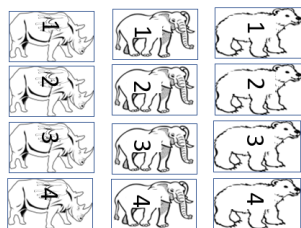

    There are 66 different two-card hands possible using this card deck. How many of these two-card hands contain two different animals?
    A. 21
    B. 24
    C. 42
    D. 48
    E. 54

14. In a popular card game on Zoo Island, players receive a two-card hand at random from the deck in #13 above. What is the probability that a two-card hand chosen at random from this deck will not contain any Elephants?

    A. $\frac{2}{3}$

    B. $\frac{4}{9}$

    C. $\frac{7}{18}$

    D. $\frac{14}{33}$

    E. $\frac{55}{132}$

15. If one card is chosen at random from the deck in #13, what is the probability that the card contains an Elephant OR a 4?
    A. $\frac{1}{12}$

    B. $\frac{5}{12}$

    C. $\frac{1}{2}$

    D. $\frac{2}{3}$

    E. $\frac{7}{12}$

16. Below is a histogram for the Quiz 1 scores for Mrs. Trammel's 8ᵗʰ grade math class. Mrs. Trammel gives quizzes with 13 possible points, and students can earn whole number scores from a minimum of 0 to a maximum of 13.


Scores on Quiz 1 for Mrs. Trammel's Class
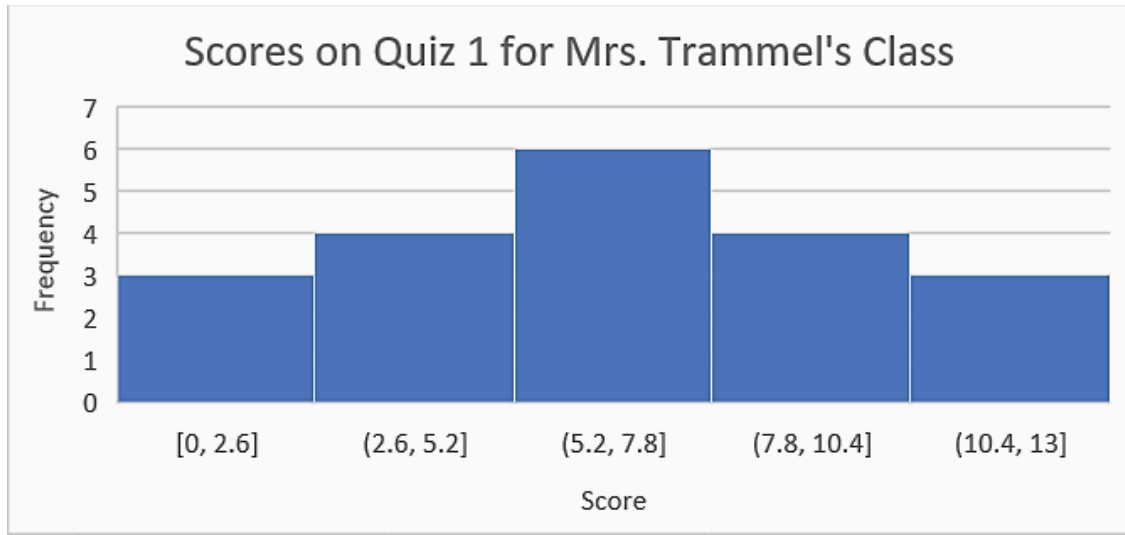
Which of the statements below must be true according to the histogram above?

I. There were 20 students taking the quiz.
II. The average score could have been below 6.
III. The mean and median score were the same.

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

17. Find the approximate area of the composite figure. The curve pictured is a semicircle.

A. 440.36 square units
B. 120.16 square units
C. 212.63 square units
D. 301.46 square units
E. 102.37 square units
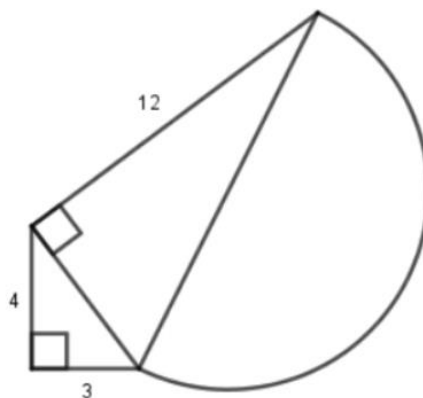
18. Mary's allowance starts out at $15 a week and increases by a constant amount of 50 cents each week. Yolanda's allowance starts out at $7 a week and also increases by a constant amount each week, but an amount different from Mary's weekly increase. After 32 weeks, Yolanda's weekly allowance will be the same as that of Mary. How much per week does Yolanda's allowance increase?
    A. $1
    B. 80 cents
    C. 75 cents
    D. 70 cents
    E. 60 cents

19. Abdul's beginning salary is $35,000, and his salary increases by 3% each year. What is his salary after t years?
    A. $35{,}000 + 1.03t$
    B. $35{,}000(1.03t)$
    C. $35{,}000(1 + .03t)$
    D. $35{,}000 + 35{,}000(0.03^t)$
    E. $35{,}000(1.03)^t$

20. A cube of side b, and a square pyramid with base b and slant height s, have the same surface area. What is the ratio of s to b?
    A. 3 to 2
    B. 4 to 3
    C. 6 to 5
    D. 5 to 2
    E. 5 to 6

21. Which of the following statements is true?
    I. The circumference of a circle and its radius can both be rational numbers.
    II. The circumference of a circle can be an integer.
    III. If the radius of a circle is an integer, its circumference is an irrational number.
    A. I only
    B. II only
    C. III only
    D. I and II only
    E. II and III only

22. Purple Paint is produced by mixing red paint with blue paint. If the perfect color of purple paint is achieved by using x times as much blue paint as red, how much red paint is needed to produce 20 gallons of perfect purple paint?

    A. $\dfrac{20}{x+1}$ gallons

    B. $\dfrac{20}{x}$ gallons

    C. $\dfrac{20}{1+\frac{1}{x}}$ gallons

    D. $20\left(1+\dfrac{1}{x}\right)$ gallons

    E. $20\left(x+\dfrac{1}{x}\right)$ gallons

23. The slope of a line with x-intercept of 5 and y-intercept of 4 is

    A. $\dfrac{4}{5}$

    B. $\dfrac{5}{4}$

    C. $-\dfrac{4}{5}$

    D. $-\dfrac{5}{4}$

    E. None of the above

24. Suppose that a is a non-negative real number. Consider the following three statements.

$$I. \quad a^2 > a \qquad II. \quad \frac{1}{a} < a \qquad III. \quad \sqrt{a} < a$$

Which of these statements is true for all non-negative real values of a?

    A. I only
    B. II only
    C. I and II only
    D. I, II, and III
    E. None of the three statements is true for all non-negative real values of a.

25. Place these three numbers in increasing order: $2048^{128}, 256^{64}, 128^{256}$

    A. $256^{64} < 128^{256} < 2048^{128}$
    B. $256^{64} < 2048^{128} < 128^{256}$
    C. $128^{256} < 256^{64} < 2048^{128}$
   D. $2048^{128} < 256^{64} < 128^{256}$
    E. $2048^{128} < 128^{256} < 256^{64}$

26. Which of the following sequences of geometric transformations will result in transforming the shaded triangle into the unshaded triangle in the figure below?

    I. Rotate 90 degrees clockwise about the point (-3,1); translate 4 units to the right; translate 2 units down.

    II. Translate 4 units to the right; reflect about the x-axis.

    III. Reflect about the line $y = x + 1$

A. I only
B. II only
C. III only
D. I and II only
E. I and III only
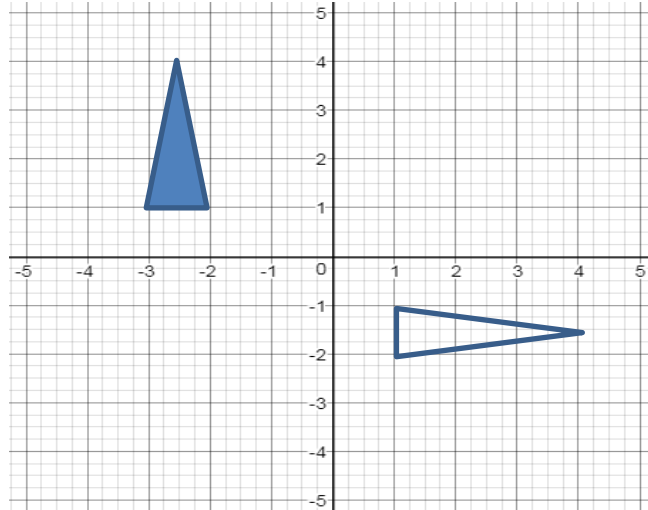
27. Angles *ACB*, *ADC*, and *BDC* are right angles. If the length of segment *AC* is 1.25 units and the length of segment *CD* is 1 unit, what percentage of the area of Δ*ACB* is triangle Δ*CDB*? (The figure is not necessarily drawn to scale.)
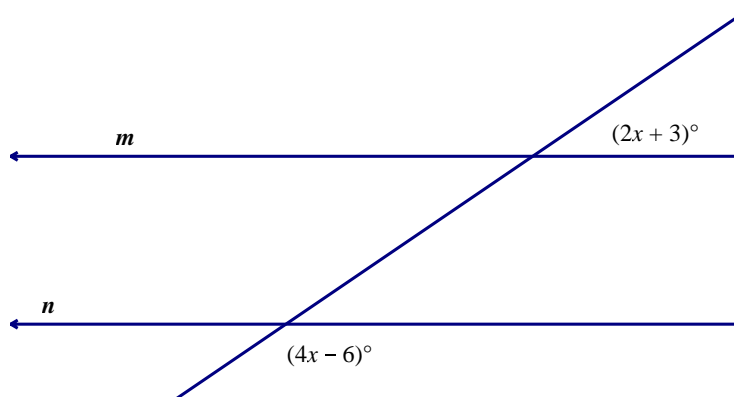
A. 90%
B. 85%
C. 80%
D. 75%
E. 64%
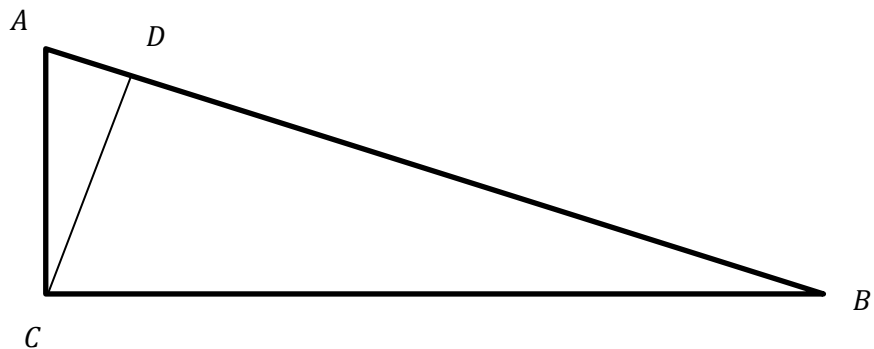
28.   For positive numbers a and b, simplify $\sqrt[4]{\sqrt{a^2 b^8}}$.
   A. $b^4\sqrt{a}$
   B. $a^4\sqrt{b}$
   C. $a\sqrt{b}$
   D. $b\sqrt{a}$
   E. $\sqrt{a\sqrt{b}}$

29. In the figure below, line m is parallel to line n.   The measures of two of the angles in the figure are (2x+3)° and (4x-6)° as labeled.   Find the value of *x*. The figure is not necessarily drawn to scale.

    A. 30.5
    B. 15.25
    C. 4.5
    D. 32.5
    E. 18.5

    m $(2x + 3)°$

    n

    $(4x − 6)°$

30. Yuri has a new cell phone. He records the battery charge for the first 20 minutes after he plugs it in.

Time charging in minutes	% charged
5 minutes	20
10 minutes	28
15 minutes	36
20 minutes	44

At this rate, how much of a charge did the phone have when it was first plugged in, and how much longer than 20 minutes will be it before the phone is fully charged?

    A. 0%; 30 minutes
    B. 0%; 35 minutes
    C. 12%; 30 minutes
    D. 12%; 35 minutes
    E. 12%; 40 minutes