

FIFTY-FIRST ANNUAL MATHEMATICS CONTEST  
sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 2007

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Scoring formula:  $4R - W + 40$

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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Contributors to TMTA for the Annual Mathematics Contest:

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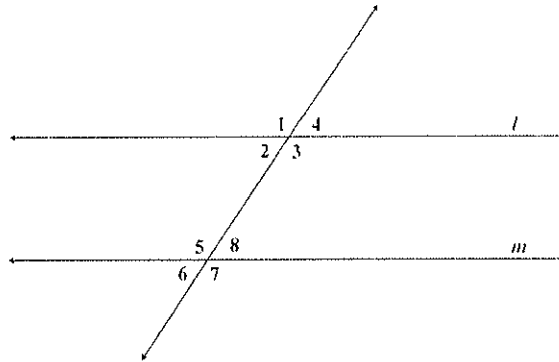
**Tennessee Mathematics Teachers' Association  
2007 Geometry Test**

1. Which of the following statements is true?

- a) Every quadrilateral is a rectangle.
- b) Every right triangle is isosceles.
- c) Every rectangle is a square
- d) Every equilateral triangle is isosceles.
- e) Every rhombus is a square.

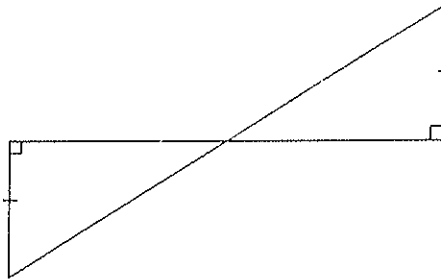
2. If  $l$  and  $m$  are parallel lines in the figure at right, which of the following pairs of angles are supplementary?

- a) 1 and 3
- b) 5 and 7
- c) 7 and 8
- d) 6 and 8
- e) 4 and 2



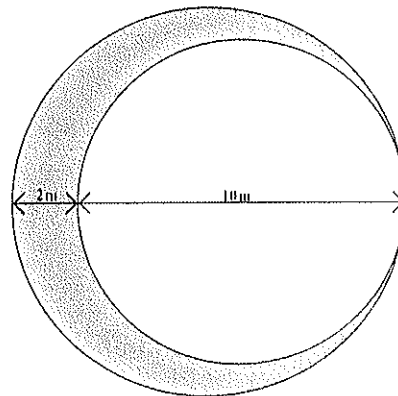
3. Can the two triangles below be proved congruent? If so, by which indicated method?

- a) SSS
- b) SAS
- c) ASA
- d) AAS
- e) Cannot be proved congruent



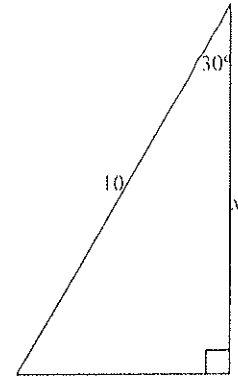
4. Find the best approximation for the area of the shaded region.

- a) 113.10 m<sup>2</sup>
- b) 34.56 m<sup>2</sup>
- c) 100.53 m<sup>2</sup>
- d) 191.64 m<sup>2</sup>
- e) 12.57 m<sup>2</sup>

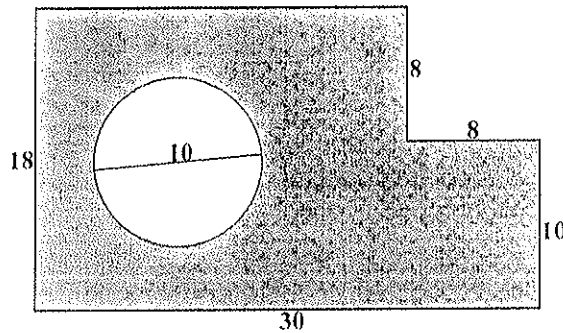


5. In the triangle shown, find the value of  $x$ :

- a) 5            b)  $5\sqrt{3}$             c)  $10 - 5\sqrt{3}$   
 d) 3            e) cannot be determined



6. Determine the area of the shaded portion of the figure. Corners are right angles.



- a) 540            b) 364            c)  $25\pi$             d)  $476 - 25\pi$             e)  $100\pi$

7. If city B is located 20 miles due northwest of city A, but the only route from A to B involves going due west to city C, and then due north to city B, what is the closest approximate length of that route, in miles?

- a) 24.16            b) 30.64            c) 28.28            d) 33            e) 36

8. The area of any triangle with sides of length  $a$ ,  $b$ , and  $c$  can be computed by the formula  $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ , where  $s =$

- a)  $\frac{1}{2}(bc + a)$   
 b)  $\frac{1}{2}(ab + c)$   
 c)  $\frac{1}{2}(a + b + c)$   
 d)  $\frac{1}{2}(a^2 + b^2)$   
 e)  $\frac{1}{2}(a^2 + b^2 - c^2)$

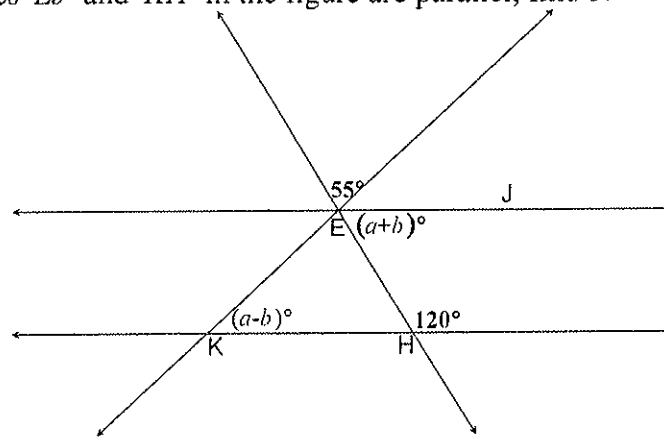
9. What is the line of reflection for a transformation which maps  $(-2,1)$  onto  $(2,1)$ ?

- a) the  $x$ -axis    b) the  $y$ -axis    c)  $y = x$             d)  $y = 1$             e) the origin

10. Find the equation of the perpendicular bisector of the segment joining the points (0,1) and (2,-3)

- a)  $y = \frac{1}{2}x + \frac{1}{2}$
- b)  $y = \frac{1}{2}x - \frac{3}{2}$
- c)  $y = 2x + 1$
- d)  $y = 2x - \frac{3}{2}$
- e)  $y = 2x + \frac{3}{2}$

11. Given that lines  $\overleftrightarrow{EJ}$  and  $\overleftrightarrow{KH}$  in the figure are parallel, find  $b$ .



- a) -2.5
- b) 62.5
- c) 60
- e) 65
- f) 52.5

12. Points A and B are 3 cm apart. How many points in the plane containing A and B are both 2 cm from A and 2 cm from B?

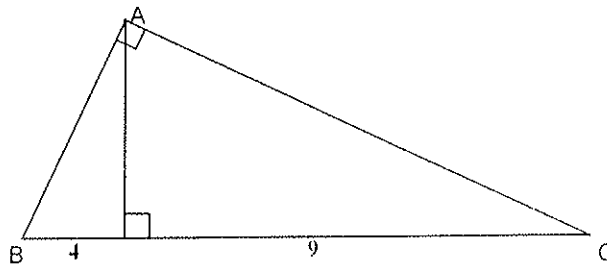
- a) 0 points
- b) 1 point
- c) 2 points
- d) 3 points
- e) infinitely many points

13. The slope of the tangent to the circle  $x^2 + y^2 = 25$  at the point (3,4) is

- a)  $-3/4$
- b)  $-5/3$
- c)  $-5/4$
- d)  $-1$
- e)  $-4/3$

14. Find the area of triangle ABC.

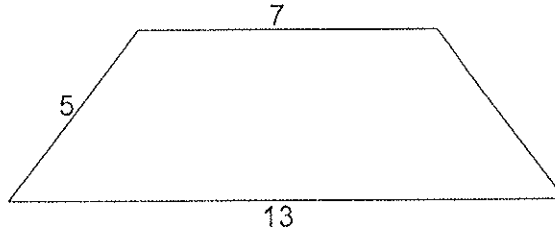
- a) 36 square units
- b) 39 square units
- c) 40 square units
- d) 48 square units
- e) 72 square units



15. The area of a circle inscribed in an equilateral triangle is  $48\pi$  square units. The perimeter of this triangle is how many units?

- a)  $72\sqrt{3}$       b)  $48\sqrt{3}$       c) 36      d) 24      e) 72

16. Find the area of this isosceles trapezoid.



- a) 35 sq units    b) 40 sq units    c) 50 sq units    d) 65 sq units    e) 100 sq units

17. A solid gold sphere with radius 3 cm is melted down and recast as a cylinder with the same radius. What is the height of the cylinder?

- a)  $\frac{1}{3}$  cm      b)  $\frac{4}{3}$  cm      c) 4 cm      d) 9 cm      e) 12 cm

18. The number of degrees in the measure of an exterior angle of a regular pentagon is

- a) 54      b) 108      c) 72      d) 112      e) 36

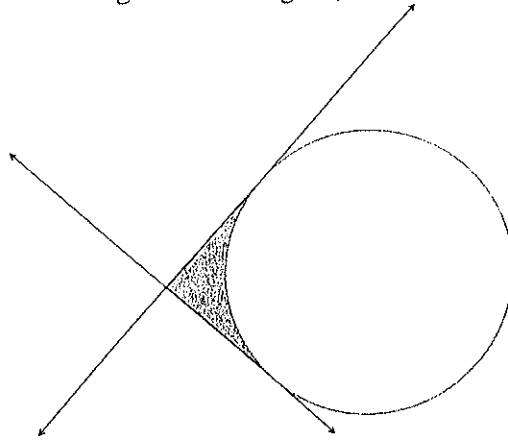
19. There is a comet passing near the earth, unique in that it is traveling in a straight line at 800 mph. NASA launches a probe that is supposed to impact on the comet. They launch the probe when the comet is on its closest approach to the earth of 210,000 miles. To the nearest mile per hour, how fast will the probe have to travel to catch the comet in exactly 5 days? Assume that the earth is a single point.

- a) 1862 mph    b) 1924 mph    c) 2013 mph    d) 2913 mph    e) 3848 mph

20. A right triangle has sides whose lengths form an arithmetic progression whose common difference is an integer. Which of the following could be the area of the triangle?

- a) 4      b) 12      c) 16      d) 18      e) 24

21. The circle in the figure has radius 2 in. If the lines in the figure are perpendicular to each other and are both tangent to the figure, find the area of the shaded region.



- a)  $4 - \pi$  square units
- b)  $\frac{4 - \pi}{2}$  square units
- c)  $8 - 2\pi$  square units
- d)  $4\pi - 4$  square units
- e) It cannot be determined using the given information.

22. If the length of a diagonal of a square is  $a + b$ , then the area of the square is

- a)  $(a + b)^2$
- b)  $\frac{1}{2}(a + b)^2$
- c)  $a^2 + b^2$
- d)  $\frac{1}{2}(a^2 + b^2)$
- e)  $4(a^2 + b^2)$

23. A circle and a square have the same perimeter. Then

- a) their areas are equal
- b) the area of the circle is the greater
- c) the area of the square is the greater
- d) it is not possible to determine which area is greater
- e) the diameter of the circle and the diagonal of the square are equal

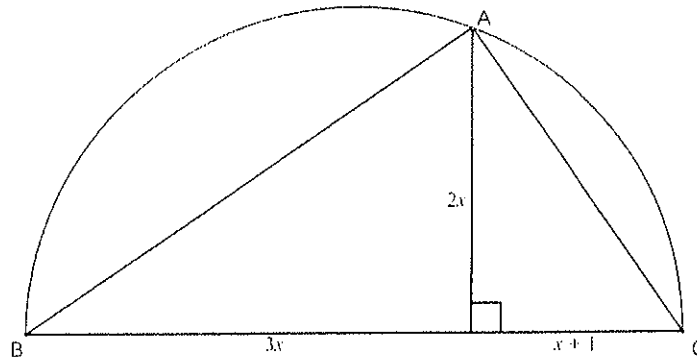
24. The Monte Carlo method can be used to find the area of a circle using probability. If dots are randomly placed in a square with sides of 2 inches each, what is the approximate probability that they fall in the inscribed circle?

- a) 0.6251
- b) 0.75
- c) 0.7854
- d) 0.8421
- e) 0.8917

25. Suppose that  $R$  is a  $3 \times 4$  rectangle. If  $A$  is the set of all points in the plane whose distance from a point inside  $R$  is no more than one, find the area of  $A$  to the nearest hundredth.

- a) 17.14
- b) 29.14
- c) 31.56
- d) 32.00
- e) 38.56

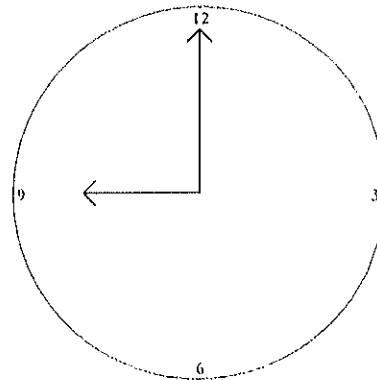
26. Given that  $\angle BAC$  is inscribed in a semicircle, find the radius of the semicircle.



- a) 3      b) 4.5      c) 6      d) 6.5      e) 9

27. When the minute hand of a clock has moved through an angle of measure  $\frac{4\pi}{3}$  radians, how many seconds will have elapsed?

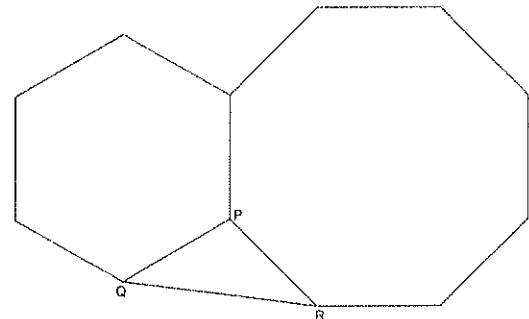
- a)  $\frac{4\pi}{3}$   
 b)  $\frac{4}{3}$   
 c) 40



- d) 2400  
 e) 500

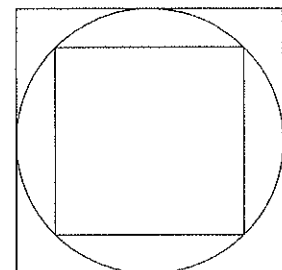
28. If the two nontriangular polygons in the figure are regular, then the measure of angles P, Q, and R in the triangle (in degrees) are

- a) 105, 37.5, 37.5  
 b) 90, 60, 30  
 c) 60, 60, 60  
 d) 120, 30, 30  
 e) 100, 60, 20



29. Find the ratio of the areas of the squares that circumscribe and inscribe a circle.

- a)  $\sqrt{2}$       b)  $\frac{\pi}{\sqrt{2}}$       c)  $\frac{\pi}{2}$       d) 2      e)  $\frac{4}{\pi}$



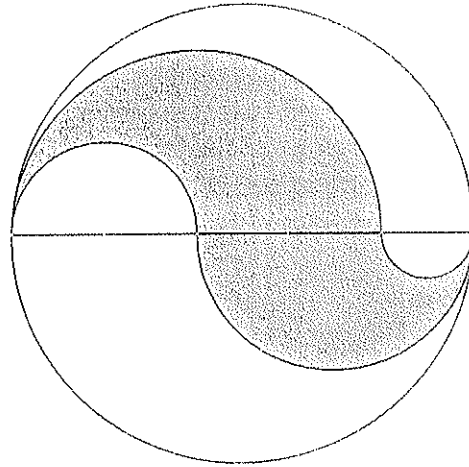


30. Circle A has a radius of 3. Circle B has a diameter  $\frac{2}{3}$  of the diameter of A, and is internally tangent to Circle A. What is the distance from the center of Circle A to the closest point of Circle B?

- a) 0.5      b) 0.75      c) 1.0      d) 1.25      e) 1.5

31. The diameter of a circle is divided into five equal parts, and semicircles are drawn on the diameter as shown. What fraction of the area of the circle is shaded?

- a)  $\frac{1}{3}$       b)  $\frac{3}{8}$       c)  $\frac{2}{5}$   
 d)  $\frac{1}{2}$       e)  $\frac{3}{7}$



32. Two vertical telephone poles, 40 feet high and 60 feet high, are placed near each other. As partial support, a cable is run from the top of each pole to the base of the other pole. How far above the level ground is the point of intersection of the two support cables?

- a) 8 feet      b) 12 feet      c) 16 feet      d) 21 feet      e) 24 feet

33. The perimeter of a regular polygon with  $n$  sides inscribed in a circle of radius  $r$  and diameter  $d$  is

- a)  $2nd \sin(\pi / n)$   
 b)  $4nr \sin(2\pi / n)$   
 c)  $2nr \sin(\pi / n)$   
 d)  $nr^2 \cos(\pi / n)$   
 e)  $nr^2 \sin(\pi / n)$

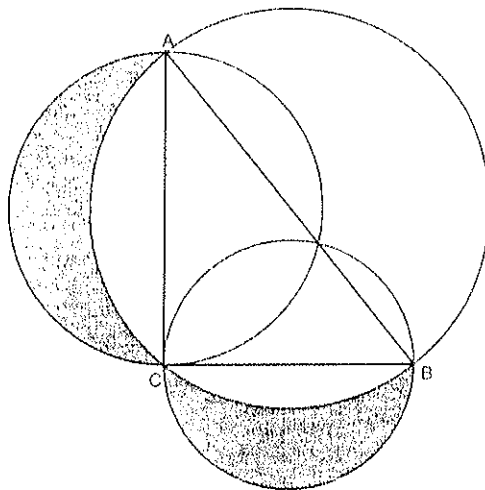
34. The ADAA wishes to increase the surface area of the official ADAA approved spherical dodgeball by 1%. By approximately what percent will the volume increase?

- a) 1      b) 1.25      c) 1.5      d) 2      e) 3

35. The diameter of a circle is divided into  $n$  equal parts. On each part a semicircle is constructed. As  $n$  becomes very large, the sum of the lengths of the arcs of the semicircles approaches a length

- a) equal to the semi-circumference of the original circle
- b) equal to the diameter of the original circle
- c) greater than the diameter but less than the semi-circumference of the original circle
- d) that is infinite
- e) finite, but greater than the semi-circumference of the original circle

36. Circles are constructed on each of the three sides of right triangle  $ABC$  using the sides as diameters. In each case, the center of the circle is the midpoint of the side, with the side being a diameter of the circle. If the area of the triangle is 12 square units, what is the total area of that part of the two smaller circles that lies outside the largest circle and is shaded in the figure?



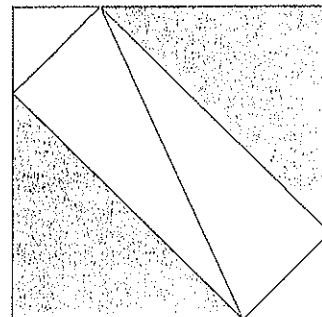
- a)  $3\pi$
- b) 9
- c) 12
- d)  $4\pi$
- e)  $5\pi$

37. Consider a figure that is constructed from two identical squares with sides of length 2. These squares lie in the same plane. One is placed exactly on top of the other, and is then rotated around the center through an angle of  $45^\circ$ . Give the closest approximate length of a side of the resulting 8-pointed star.

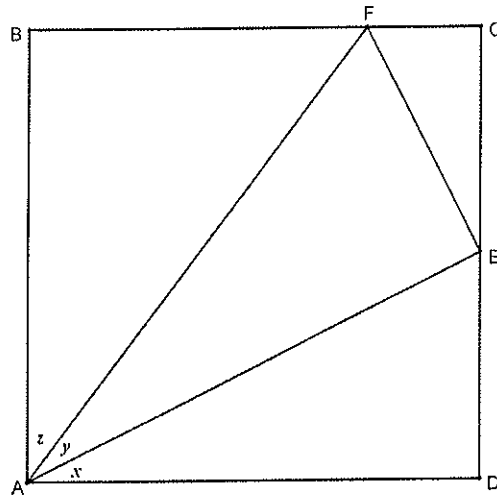
- a) 0.207
- b) 0.500
- c) 0.586
- d) 0.602
- e) 0.707

38. A rectangle is inscribed in a square such that there is an isosceles triangle in each corner. The combined area of the four triangles is 200. Find the length of the rectangle's diagonal.

- a) 20
- b) 25
- c) 30
- d) 35
- e) 40



39. ABCD is a square with E the midpoint of CD. AE is perpendicular to EF. If  $x$ ,  $y$ , and  $z$  are the measures of the angles EAD, FAE, and BAF, respectively, then the relationships among these measures are:



- a)  $x = y = z$     b)  $x > y > z$     c)  $x < y < z$     d)  $z > x = y$     e)  $z < x = y$

40. A  $6 \times 8$  piece of paper is folded so that opposite vertices touch. How long is the fold?

- a) 10 units                      b) 7 units  
 c)  $\frac{17}{2}$  units                  d)  $\frac{15}{2}$  units  
 e)  $\frac{13}{2}$  units

