

FORTY-FIFTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 2001

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

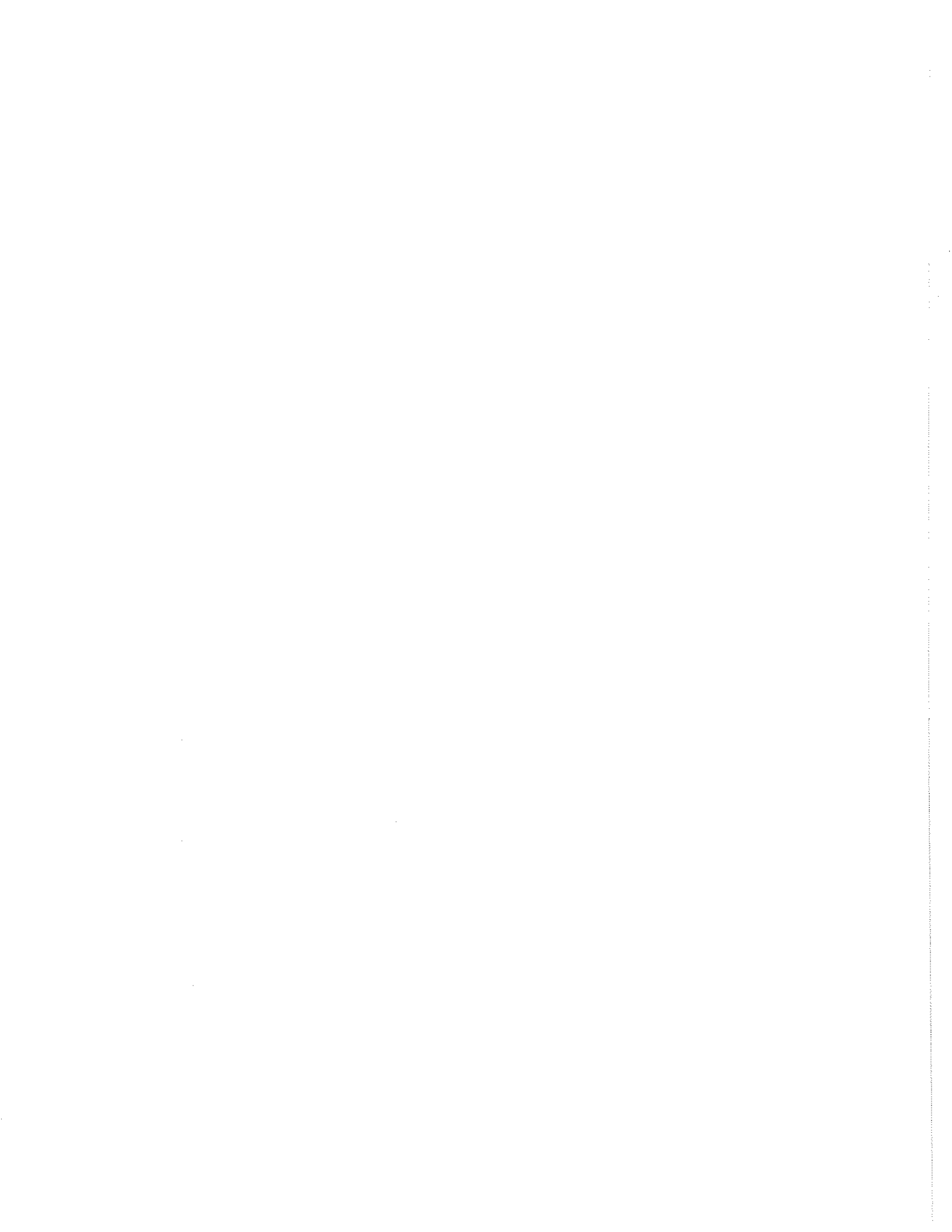
This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

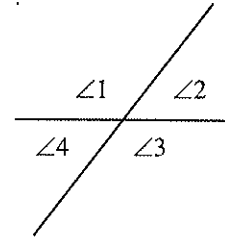
Contributors to TMTA for the Annual Mathematics Contest:

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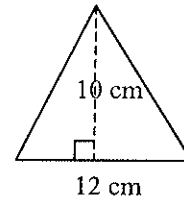
1. Which of the following pairs of angles are supplementary?
- (a) $32^\circ, 32^\circ$
 - (b) $32^\circ, 148^\circ$
 - (c) $40^\circ, 50^\circ$
 - (d) $45^\circ, -45^\circ$
 - (e) $48^\circ 30', 132^\circ 30'$

2. Which statement is incorrect, according to the diagram?
- (a) $\angle 1$ and $\angle 2$ are supplementary
 - (b) $\angle 1$ and $\angle 3$ are vertical angles
 - (c) $m\angle 2 = m\angle 4$
 - (d) $m\angle 3 + m\angle 4 = 90^\circ$
 - (e) $\angle 1 \cong \angle 3$

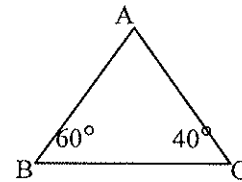


3. Which of the following sets of lengths can be the lengths of sides of a right triangle?
- (a) 2, 3, 4
 - (b) 3, 4, 5
 - (c) 5, 6, 7
 - (d) 6, 7, 8
 - (e) 8, 9, 10
4. Which of the following sets of lengths cannot be the lengths of sides of a triangle?
- (a) 3, 4, 5
 - (b) 10, 11, 12
 - (c) 12, 20, 22
 - (d) 3, 2, 49
 - (e) 5, 12, 13

5. Find the area of the triangle in the diagram.
- (a) 120 cm^2
 - (b) 120 cm
 - (c) 60 cm
 - (d) 60 cm^2
 - (e) 60



6. Find the measure of $\angle A$, according to the diagram.
- (a) 80°
 - (b) 100°
 - (c) 90°
 - (d) 70°
 - (e) 20°

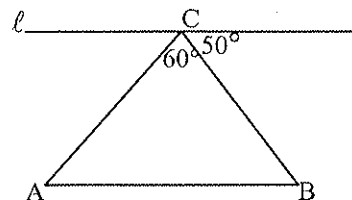


7. Which of the following is not a triangle congruence theorem?
- (a) SSS
 - (b) HL
 - (c) SSA
 - (d) ASA
 - (e) AAS
8. Find the exact area of a circle with radius 3 in.
- (a) 28.26 in^2
 - (b) 9.42 in^2
 - (c) $3\pi \text{ in}^2$
 - (d) $9\pi \text{ in}^2$
 - (e) $6\pi \text{ in}^2$

9. What is the name of a polygon with ten sides?
- dodecagon
 - decagon
 - nonagon
 - dodecahedron
 - Bob
10. Which of the following figures are not also parallelograms?
- squares
 - rhombuses
 - rectangles
 - trapezoids
 - regular quadrilaterals
11. Which of the following cannot happen in an isosceles triangle?
- base angles are congruent
 - at least two sides are equal in length
 - at least one angle is obtuse
 - the base is longer than either of the two congruent sides
 - a base angle is a right angle
12. Only one of the following represents correct notation. Which is it? You should assume that all points are distinct.
- $\overline{AB} \cong \overline{CD}$
 - $\angle ABC \cong 90^\circ$
 - $AB \cong CD$
 - $\angle A \perp \angle B$
 - $\overline{AB} = \overline{CD}$
13. Find the difference of the sum of the interior angles and the sum of the exterior angles of a convex pentagon.
- 0°
 - 180°
 - 360°
 - 540°
 - 900°

14. Find the measure of $\angle A$, according to the diagram, if $\ell \parallel \overline{AB}$.

- 60°
- 70°
- 80°
- 90°
- 110°

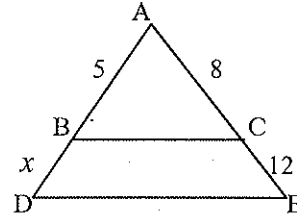


15. How many edges does an octahedron have?

- (a) 4
- (b) 6
- (c) 8
- (d) 12
- (e) 20

16. Find x according to the diagram, if $\overline{BC} \parallel \overline{DE}$.

- (a) 7.5
- (b) 12
- (c) 9
- (d) 8
- (e) 15



17. A cube has volume 60 m^3 . A second cube has sides whose lengths are half that of the first cube. What is the volume of the second cube?

- (a) 7.5 m^3
- (b) 30 m^3
- (c) 15 m^3
- (d) 120 m^3
- (e) 480 m^3

18. The interior of a regular polygon is divided into four triangular regions by drawing diagonals from one vertex. How many sides must the polygon have?

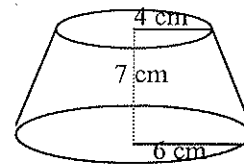
- (a) 4
- (b) 5
- (c) 6
- (d) 8
- (e) 9

19. The sides of a triangle have length 6 cm, 8 cm and 9 cm. Then

- (a) the triangle is acute
- (b) the triangle is obtuse
- (c) the angles of the triangle are in the ratio 6:8:9
- (d) the triangle is isosceles
- (e) the area of such a triangle cannot be found

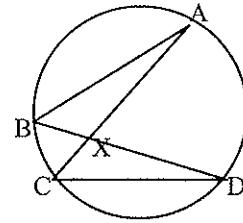
20. A frustrum of a right circular cone has radii 4 cm and 6 cm with height 7 cm, as pictured. What is its volume?

- (a) $364 \pi \text{ cm}^3$
- (b) $\frac{532}{3} \pi \text{ cm}^3$
- (c) $175 \pi \text{ cm}^3$
- (d) $84 \pi \text{ cm}^3$
- (e) $\frac{140}{3} \pi \text{ cm}^3$



21. In the diagram, $m\angle AXD = 80^\circ$ and $m\angle XDC = 20^\circ$. Find $m\angle ABX$.

- (a) 50°
- (b) 20°
- (c) 80°
- (d) 100°
- (e) 60°



22. A rectangular solid has sides of length 3 in, 4 in and 12 in. What is the length of its diagonal?

- (a) 5 in
- (b) $3\sqrt{17}$ in
- (c) $4\sqrt{10}$ in
- (d) 13 in
- (e) $4\sqrt{10}$ in

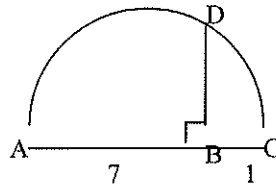
23. A square is inscribed in a circle of radius 1 m. What is the area of the square?

- (a) 2 m^2
- (b) $\pi \text{ m}^2$
- (c) $2\pi \text{ m}^2$
- (d) $2\sqrt{2} \text{ m}^2$
- (e) $\sqrt{5} \text{ m}^2$

24. In the diagram, $AB = 7$, $BC = 1$, $m\angle ABD = 90^\circ$, and the arc is a semicircle.

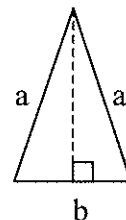
Find $AD + DC$

- (a) $\sqrt{7}$
- (b) $2\sqrt{2}$
- (c) $2\sqrt{14}$
- (d) $2(\sqrt{2} + \sqrt{14})$
- (e) 8



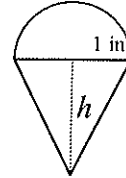
25. In the isosceles triangle pictured, $a > b$. Put the orthocenter (intersection of the altitudes), circumcenter (intersection of the perpendicular bisectors of the sides of the triangle), and centroid (intersection of medians) in order from top to bottom as they appear on the dotted line.

- (a) orthocenter, centroid, circumcenter
- (b) centroid, orthocenter, circumcenter
- (c) centroid, circumcenter, orthocenter
- (d) circumcenter, centroid, orthocenter
- (e) circumcenter, orthocenter, centroid



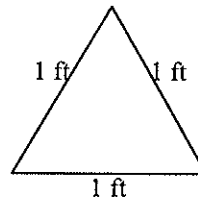
26. An ice cream cone consists of a right circular cone filled with ice cream topped by a half sphere of ice cream. The radius of the sphere is 1 in. What value for h would make the volume of ice cream inside the cone equal to that of the half sphere?

- (a) 1 in
- (b) 2 in
- (c) 3 in
- (d) π in
- (e) 4 in



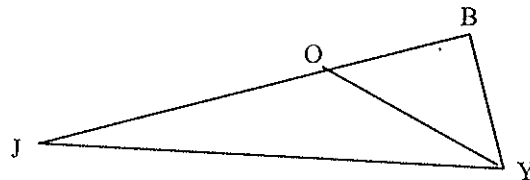
27. An equilateral triangle oriented as in the diagram whose sides measure 1 ft is painted from bottom to top, beginning at its base. At what height will half the triangle (by area) be painted?

- (a) $\frac{\sqrt{3}}{2}$ ft
- (b) $\frac{(2-\sqrt{2})\sqrt{3}}{4}$ ft
- (c) $\frac{1}{2}$ ft
- (d) $\frac{\sqrt{6}}{4}$ ft
- (e) $\frac{\sqrt{3}}{4}$ ft



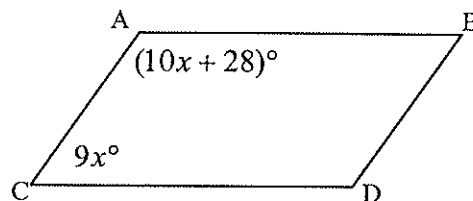
28. Find the area of triangle JOY given $JO = 8$ mm, $BY = 3\sqrt{2}$ mm, $m\angle JBY = 90^\circ$ and $m\angle BOY = 45^\circ$.

- (a) $9\sqrt{7}$ mm²
- (b) $\sqrt{455}$ mm²
- (c) $9\sqrt{7} - 9$ mm²
- (d) $12\sqrt{2}$ mm²
- (e) not enough information given



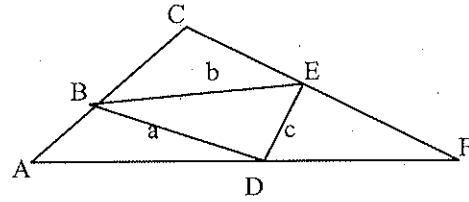
29. Find $m\angle D$ given that ABDC is a parallelogram, $m\angle A = (10x + 28)^\circ$, and $m\angle C = 9x^\circ$.

- (a) 8°
- (b) 72°
- (c) 108°
- (d) 78°
- (e) 134°



30. Given $AB = 2$, $BC = 2$, $CE = 3$, $EF = 4$, $DF = 5$, and $AD = 5$, which of the following values is correct?

- (a) $a = 5$
- (b) $c = 2$
- (c) $a = 3.5$
- (d) $c = 4$
- (e) $b = 5$



31. Which of the following does this statement define? "Given two distinct points A and B, _____ is the set containing A, B, and all points C such that either C is between A and B or B is between A and C."

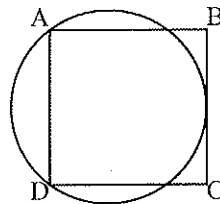
- (a) the segment \overline{AB}
- (b) the ray \overrightarrow{AB}
- (c) the line \overleftrightarrow{AB}
- (d) the circle centered at A with radius AB
- (e) the ray \overrightarrow{BA}

32. Which statement is false?

- (a) No angle can be trisected using compass and straightedge only.
- (b) There does exist an angle that can be trisected using compass and straightedge only.
- (c) There is a method to trisect an arbitrary angle, but not with compass and straightedge only.
- (d) Any angle can be bisected using compass and straightedge only.
- (e) Any line segment can be trisected using compass and straightedge only.

33. The length of each side of the square in the diagram is 8 ft. A circle is drawn through A and D tangent to \overline{BC} . What is the radius of the circle?

- (a) 4 ft
- (b) $4\sqrt{2}$ ft
- (c) 5 ft
- (d) $5\sqrt{2}$ ft
- (e) 6 ft

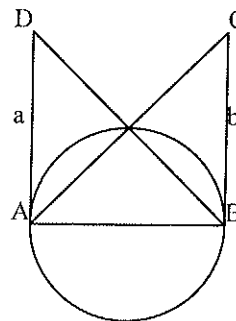


34. A province's northern and southern borders lie along lines of latitude while its eastern and western borders lie along lines of longitude. Then

- (a) its northern border is longer than its southern border
- (b) its southern border is longer than its northern border
- (c) its northern and southern borders have the same length
- (d) the relationship between its northern and southern borders can be determined only if we know in which hemisphere (northern or southern) the province lies
- (e) its eastern and western borders are equal in length only if the equator passes through the province

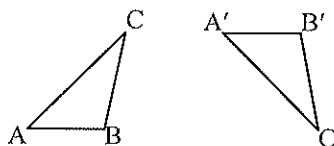
35. In the figure, \overline{AB} is a diameter of the circle; \overline{AD} and \overline{BC} are tangents drawn so that \overline{AC} and \overline{BD} intersect at a point on the circle. Let $a = AD$ and $b = BC$, and suppose that $a < b < 2a$. Find the diameter of the circle.

- (a) $|a - b|$
 (b) $\frac{a + b}{2}$
 (c) \sqrt{ab}
 (d) $\frac{ab}{a + b}$
 (e) $\frac{ab}{2(a + b)}$



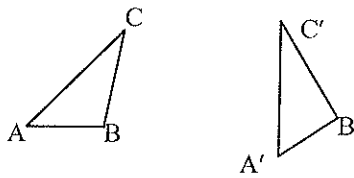
36. The motion of the plane that takes $\triangle ABC$ to $\triangle A'B'C'$ is a(n):

- (a) translation
 (b) rotation
 (c) inversion
 (d) reflection
 (e) glide reflection



37. The motion of the plane that takes $\triangle ABC$ to $\triangle A'B'C'$ is a(n):

- (a) translation
 (b) rotation
 (c) inversion
 (d) reflection
 (e) glide reflection



38. Let S be the sum of the measures of the angles of a triangle in hyperbolic geometry. Which statement best represents the truth?

- (a) $S < 180^\circ$ (b) $S \leq 180^\circ$ (c) $S = 180^\circ$ (d) $S \geq 180^\circ$ (e) $S > 180^\circ$

39. A Möbius strip is cut in half, down its center line. What is the result?

- (a) two Möbius strips
 (b) two regular loops
 (c) one Möbius strip, twice as long as the original Möbius strip
 (d) a two-sided strip, twice as long as the original Möbius strip
 (e) a long, thin rectangle

40. Which of the following is not a Platonic solid?

- (a) A regular tetrahedron
 (b) A regular octahedron
 (c) A regular decahedron
 (d) A regular dodecahedron
 (e) A regular icosahedron

