FIFTIETH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Calculus and Advanced Topics 2006

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Scoring formula: 4R − W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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1. In decomposing $\frac{2x+19}{(2x-1)(x+2)}$ by partial fraction decomposition, one of the rational fractions obtained is:

a) $\frac{-3}{x+2}$ b) $\frac{-6}{2x-1}$ c) $\frac{3}{x+2}$ d) $\frac{6}{2x-1}$ e) $\frac{-6}{x+2}$

2. Given the greatest integer function $\|x\|$, if $f(x) = \|x\| + \|-x\|$, then $\lim_{x \to a} f(x)$ exists for what values of a?

a) for all reals
b) does not exist anywhere
c) the integers
d) all non-integers
e) for all rational numbers

3. If $y = x^3 \cdot h(x)$, then $y' =$

a) $3x^2 \cdot h(x)$
b) $3x^2 \cdot h(x) - x^3 \cdot h'(x)$
c) $3x^2 \cdot h(x) + x^3 \cdot h'(x)$
d) $3x^2 \cdot h'(x)$
e) $x^3 \cdot h(x)$

4. $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} =$

a) 0 b) 1 c) $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}$ d) π e) the limit does not exist

5. Which of the following has a removable discontinuity at a?

a) $\frac{x^2-2x-8}{x+2}$, a = −2
b) $\frac{x-7}{|x-7|}$, a = 7
c) $\frac{x^3+64}{x-4}$, a = 4
d) $\frac{\sin x}{\cos x}$, a = $\frac{\pi}{2}$
e) $\frac{3+\sqrt{x}}{9-x}$, a = 9

6. The points on $x^2y^2 + xy = 2$ where the slope of the tangent line to the curve at those points is equal to -1 are given by the relation

a) $y = -x$ b) $y = 2x$ c) $y = \frac{1}{x}$ d) $y = x^2$ e) $y = x$

7. $\frac{d}{dx} f(4x^3) =$

a) $12x^2 f(4x^3)$ b) $12x^2 f'(4x^3)$ c) $4x^3 f(x)$ d) $f'(4x^3)$ e) $12x^2 + f'(4x^3)$

8. If f is differentiable and $a > 0$, then write $\lim_{x \to a} \frac{f(x)-f(a)}{\sqrt{x}-\sqrt{a}}$ in terms of a and $f'(a)$.

a) $\sqrt{a} f'(a)$
b) $\frac{1}{2}\sqrt{a} f'(a)$
c) $af'(a)$
d) $2\sqrt{a} f'(a)$
e) $\sqrt{af'(a)}$

9. Let $f(x)=\begin{cases} x^2, & x \le 2 \\ mx+b, & x > 2 \end{cases}$. The values of m and b, respectively, that make f differentiable everywhere are:

a) 2 and 2 b) 4 and 2 c) 4 and −4 d) 2 and 0 e) 4 and 0

10. $\sum_{k=0}^{\infty}\left(\frac{3}{4}\right)^k =$

a) $\left(\frac{3}{4}\right)^2$ b) 2 c) 4 d) diverges e) 1

11. For what values of p does $\int_2^{\infty}\frac{dx}{x^p \ln x}$ converge?

a) $p < 1$ b) $p \le 1$ c) $p > 1$ d) $p \ge 1$ e) diverges for all p

12. If $f(x) = \int_a^x e^t L(t)dt$ where L(t) is a continuous function over the interval $[a, b]$ containing x, then $f'(x) =$

a) L(x)

b) $e^x \cdot L'(x) + e^x \cdot L(x)$

c) $e^x \cdot L(x)$

d) $e^x L'(x)$

e) cannot be determined

13. In how many ways can a committee of 4 women and 3 men be chosen from a group of 10 women and 10 men?

a) 25,200 b) 3,628,800 c) $4! \cdot 3!$ d) 4!+3! e) 700

14. $\sum_{k=0}^{\infty}\frac{x^k}{k!}$ has interval of convergence

a) $(-\infty,\infty)$ b) (-1, 1) c) $(0,\infty)$ d) $\left(-\frac{1}{k},\frac{1}{k}\right)$ e) diverges for all x

15. A particular softball player has a batting average of .300 (3 hits in 10 at bats on the average). What is the approximate probability that she gets at least 3 hits in 5 at bats?

a) .132 b) .162 c) .97 d) .030 e) .838

16. $\int \sin(2x)dx =$

a) $-\frac{1}{2}\cos(2x)+C$

b) $\frac{\tan(2x)}{2}+C$

c) $-\cos(2x) + C$

d) $\frac{\sec(2x)}{2}+C$

e) $\frac{-\cos(2x)}{2}$

17. How many different pizzas can be made if the possible toppings are cheese, mushroom, pepperoni, sausage, beef, and bacon? (You must use at least one topping and each topping can be used only once.)

a) 6! B) 21 c) 1956 d) 64 e) 63

18. $\lim_{h \to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} =$

a) $\frac{1}{2}x$ b) $\frac{1}{2\sqrt{x}}$ c) $\frac{1}{2}\sqrt{x}$ d) $2\sqrt{x}$ e) $\sqrt{x}$

19. The volume of the region obtained by revolving the area between $y = x^2$ and $y = 1$ around the line $y = 2$ is:

a) $\frac{8\pi}{15}$ b) $\frac{\pi}{5}$ c) $\frac{8\pi}{5}$ d) 8π e) $\frac{56\pi}{15}$

20. $\int xe^{2x}dx =$

a) $\frac{x^2}{4}e^{2x} + C$

b) $2xe^{2x} + e^{2x} + C$

c) $\frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + C$

d) $\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$

e) $\frac{xe^{2x}}{2} + \frac{e^{2x}}{2} + C$

21. $h(1) = -2,\ h'(1) = 2,\ h''(1) = 3,\ h'(2) = 5,\ h''(2) = 13,\ h''$ is continuous everywhere.

$$\int_1^2 h''(u)\,du =$$

a) 3
b) 10
c) 8
d) 4
e) not enough information to determine the answer.

22. The function $f(x) = \begin{cases} kx^2 - x, & x < 2 \\ 3x - 1, & 2 \le x \end{cases}$ is continuous at $x = 2$ if k has the value

a) $\frac{7}{4}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) $\frac{9}{4}$ e) 2

23. In finding areas under the normal curve, if we wish to determine the area between *A* and *B*, and *A* is less than the mean while *B* is greater than the mean, we find the area between the mean and *A* and then

a) subtract the area between the mean and *B*

b) add the area between the mean and *B*

c) subtract it from .5

d) add it to .5

e) subtract it from .5, then add to the area between the mean and *B*

24. Suppose that $g(v)$ is the amount of drag the air is exerting on an aircraft traveling at a velocity of v. If $\frac{dg}{dv} > 0$ for $0 < v <$ *speed of sound*, then the drag caused by air friction

a) decreases with an increase of velocity for $v <$ *speed of sound.*

b) stays the same with an increase of velocity for $v <$ *speed of sound.*

c) cannot be analyzed with the information given.

d) increases with an increase of velocity for $v <$ *speed of sound.*

e) varies periodically.

25. Let f be one-to-one, differentiable, and $g = f^{-1}$. If $f(0) = 1$, $f'(0) = 2$, and $f'(1) = 1$, then $g'(1) =$

a) 0 b) -1 c) 1 d) $-\frac{1}{2}$ e) $\frac{1}{2}$

26. For $f(x) = (x-1)(x+2)^2(x-3)^3$, $\frac{f'(x)}{f(x)} =$

a) $\frac{1}{x-1} \cdot \frac{2}{x+2} \cdot \frac{3}{x-3}$

b) $\frac{1}{x-1} + \frac{2}{(x+2)^2} + \frac{3}{(x-3)^3}$

c) $\frac{1}{x-1} + \frac{2}{x+2} + \frac{3}{x-3}$

d) $\frac{1}{x-1} \cdot \frac{2}{(x+2)^2} \cdot \frac{3}{(x-3)^3}$

e) $\frac{3}{x-1} + \frac{2}{x+2} + \frac{1}{x-3}$

27. $\int_0^{\pi}\left[\int_0^1 x^2 \sin(y)\,dx\right]dy$

a) $\frac{2}{3}$ b) 2 c) $\frac{1}{3}$ d) 3 e) $\frac{\pi}{4}$

28. Let $f(x,y) = xy^2 + e^x$. Then $\frac{\partial f}{\partial y} =$

a) $y^2 + e^x$ b) $x + e^x$ c) xy^2 d) $2xy$ e) $2xyy'$

29. A homogenous system of n-equations and n-unknowns has a coefficient matrix A whose determinant is nonzero. The system has

a) no solution

b) an infinite number of solutions

c) only the trivial solution

d) possible " a" or "b"

e) not enough information to determine the number of solutions

30. Let f be a function and x_0, x_1, x_2, x_3, x_4 be given values such that $f(x) > 0$, continuous on $x_0 \le x \le x_4$, and let $x_0 < x_1 < x_2 < x_3 < x_4$. Let f be increasing on (x_0, x_1), (x_2, x_3) and decreasing on (x_1, x_2), (x_3, x_4). f is concave down on (x_0, x_1), (x_3, x_4) and concave up on (x_1, x_3). On which interval is $f'(x) > 0$ and $f''(x) < 0$?

a) (x_0, x_1) b) (x_1, x_2) c) (x_2, x_3) d) (x_3, x_4) e) (x_1, x_3)

31. Let $f(x,y) = xy^2 + e^x$. Then $\nabla f(0, 1) =$

a) $2\mathbf{i}$ or $\langle 2,0\rangle$

b) $\mathbf{j}$ or $\langle 0,1\rangle$

c) $2\mathbf{i} + \mathbf{j}$ or $\langle 2,1\rangle$

d) $2\mathbf{i} - \mathbf{j}$ or $\langle 2,-1\rangle$

e) $2\mathbf{i} + 2\mathbf{j}$ or $\langle 2,2\rangle$

32. The equation of the plane tangent to the surface $z = 2x^2 + y^2$ at the point (1, 1, 3) is:

a) $4x - 2y + z = 4$

b) $2x + 4y - z = 3$

c) $4x + 2y - z = 3$

d) $2x - 4y + z = 1$

e) $4x + 4y - z = 1$

33. The set $\left\{\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right\}$ is a

a) linearly independent set

b) linearly dependent set

c) linearly reliable set

d) trifold set

e) generating set for ^{3}

34. Let $A = \begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$. Then if $Ax = \lambda x$ the values of λ and x are:

a) $\lambda = 1$ and $x = \begin{pmatrix}1\\0\end{pmatrix}$.

b) $\lambda = -1$ and $x = \begin{pmatrix}0\\1\end{pmatrix}$.

c) $\lambda = -1$ and $x = \begin{pmatrix}1\\0\end{pmatrix}$.

d) $\lambda = 1$ and $x = \begin{pmatrix}0\\3\end{pmatrix}$.

e) $\lambda = -1$ and $x = \begin{pmatrix}0\\3\end{pmatrix}$.

35. Given the system

$$\begin{aligned} x_1 - x_2 &= -2 \\ -2x_1 + x_2 &= 1 \end{aligned},$$

the solution in ^{2} is

a) $\begin{pmatrix}1\\3\end{pmatrix}$ b) $\begin{pmatrix}2\\2\end{pmatrix}$ c) $\begin{pmatrix}2\\5\end{pmatrix}$ d) $\begin{pmatrix}3\\5\end{pmatrix}$ e) $\begin{pmatrix}-1\\3\end{pmatrix}$

36. If $A = \begin{pmatrix}2 & 0\\-1 & 3\end{pmatrix}$ and $B = \begin{pmatrix}-1 & 3 & 1\\7 & 0 & 2\end{pmatrix}$, then $AB =$

a) 7 b) $\begin{pmatrix}-2 & 0\\-7 & 0\end{pmatrix}$ c) $\begin{pmatrix}-2 & 6 & 2\\22 & -3 & 5\end{pmatrix}$ d) incalculable. e) $\begin{pmatrix}-2\\-7\end{pmatrix}$

37. If $Av = 0$ then v is said to be in the

a) range of A

b) null space of A

c) eigenspace of A

d) neighborhood of zero

e) hyperspace

38. The truth table for "P and not Q", also denoted in symbolic form "$P \wedge \sim Q$" is given by

a)

P	Q	$P \wedge \sim Q$
T	T	T
T	F	T
F	T	F
F	F	F

b)

P	Q	$P \wedge \sim Q$
T	T	T
T	F	F
F	T	F
F	F	F

c)

P	Q	$P \wedge \sim Q$
T	T	T
T	F	F
F	T	F
F	F	T

d)

P	Q	$P \wedge \sim Q$
T	T	F
T	F	T
F	T	F
F	F	F

e)

P	Q	$P \wedge \sim Q$
T	T	F
T	F	F
F	T	T
F	F	F

39. Some people are deceitful. No student is deceitful. Therefore we can conclude

a) Some people are students.

b) All people are not students.

c) Some people are not students.

d) Some students are deceitful.

e) Some students are not people.

40. In general, at least one solution to a linear equation is real and at least one solution to a cubic equation is real. No solution to our equation is real. Therefore, we can conclude

a) our equation may be cubic.

b) our equation is not cubic and is not linear.

c) our equation is not cubic or it is not linear.

d) our equation may be linear.

e) our equation is cubic.