

FORTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 2004

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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Algebra II

1. An equation of the line through the point $(2,-3)$ perpendicular to $3x - y = -8$ is:

- a. $3x - y = 9$
- b. $3x + y = 3$
- c. $x - 3y = 11$
- d. $x + 3y = -7$
- e. $2x - 3y = 13$

2. Factor completely $64x^3 - 216$

- a. $(4x - 6)(16x^2 + 24x + 36)$
- b. $(2x + 3)(32x^2 - 48x + 72)$
- c. $8(2x - 3)(4x^2 + 6x + 9)$
- d. $(4x^2 - 6)(16x + 36)$
- e. $4(4x + 6)(4x^2 - 6x + 9)$

3. $(3x - 4)^2 =$

- a. $9x^2 - 24x + 16$
- b. $9x^2 + 16$
- c. $9x^2 - 16$
- d. $9x^2 - 12x + 16$
- e. $9x^2 + 12x - 16$

4. The solution set for $|3x + 2| = 7$ is

- a. $\left\{\frac{-5}{3}, 3\right\}$
- b. $\left\{1, -\frac{1}{3}\right\}$
- c. $\left\{\frac{5}{3}, -3\right\}$
- d. $\left\{-1, \frac{1}{3}\right\}$
- e. $\{2, 1\}$

5. A fair die is rolled. The probability of getting a 1 or a 2 is

- a. 2
- b. $\frac{1}{6}$
- c. $\frac{1}{9}$
- d. $\frac{1}{3}$
- e. $\frac{1}{2}$

6. Which one of the following is true?

- a. $(3+7i)(3-7i)$ is an imaginary number.
- b. $\frac{7+3i}{5+3i} = \frac{7}{5}$
- c. In the complex number system $x^2 + y^2$, the sum of two squares, can be factored as $(x + yi)(x - yi)$.
- d. The conjugate of $7 + 3i$ is $-7 - 3i$.
- e. $i^{-1} = i$

7. The sum of the zeros of the polynomial $3x^2 - bx - 10$ is

- a. $-\frac{10}{3}$
- b. $\frac{b}{3}$
- c. -7
- d. $-7 - b$
- e. $-\frac{1}{3}$

8. When $6x^4 + 5x^3 + 3x - 5$ is divided by $3x^2 - 2x$, the remainder is

- a. $9x$
- b. 5
- c. $7x - 5$
- d. $2x^2 + 3x + 2$
- e. $-x + 5$

9. Given that $1 + 4 + 9 + \dots + 81 = 285$, then $2 + 11 + 26 + \dots + 242 =$
- 826
 - 872
 - 892
 - 846
 - 914
10. Tickets to a movie cost \$22 for two adults and one child. They cost \$21 for one adult and three children. What is the cost for 1 adult and 1 child?
- \$10
 - \$11
 - \$12
 - \$13
 - \$14
11. If x varies directly with y^2 , and $x = 7$ when $y = 6$, find x when $y = 3$.
- 1.0
 - 1.25
 - 1.75
 - 3.5
 - 4.0
12. In the drawing for a prize at a party ping-pong balls numbered 1 through 45 are dropped into a spinning drum. A ball is drawn out at random. What is the probability that the number on the ball is a multiple of 3?
- $\frac{4}{15}$
 - $\frac{1}{3}$
 - $\frac{14}{45}$
 - $\frac{7}{23}$
 - $\frac{1}{4}$

13. $f(x) = x^2 - 2x$, $g(x) = 2x + 3$. $(f \circ g)(x) =$

- a. $2x^2 - 4x + 3$
- b. $4x^2 + 10x + 9$
- c. $4x^2 - 4x + 3$
- d. $4x^2 - 4x + 15$
- e. $4x^2 + 8x + 3$

14. The solution set for the inequality $\left| \frac{3x+3}{5} \right| \geq 3$ is

- a. $\{x \mid -6 \leq x \leq 4\}$
- b. $\{x \mid x \geq 4 \text{ or } x \leq -6\}$
- c. $\{x \mid x \geq 4\}$
- d. $\{x \mid x \leq 4\}$
- e. $\{x \mid x \geq -6\}$

15. The domain of $f(x)$, where $f(x) = \frac{3x-2}{\sqrt{9-x^2}}$, is

- a. $(-3, 3)$
- b. $[-3, 3]$
- c. $(-\infty, \infty)$
- d. $(-\infty, -3) \cup (3, \infty)$
- e. $(-\infty, -3] \cup [-3, \infty)$

16. If $f(x) = 3x^2 - 5$, find $\frac{f(x+h) - f(x)}{h}$

- a. $6x + 3h$
- b. $6x$
- c. 1
- d. $\frac{3x^2 - x - 5}{h}$
- e. $\frac{3h^2 + 5}{h}$

$$17. \frac{27a^3b^{-3}}{-(3^{-1}a^2b^3c)^{-2}} =$$

- a. $-3a^7b^2c^2$
- b. $-3a^7b^3c^2$
- c. $3a^7b^3c^2$
- d. $3a^7b^2c^2$
- e. $\frac{-3a^7b^3}{c^2}$

18. Find the value of a in the quadratic equation $y = ax^2 + bx + c$, if its graph passes through $(1,5)$, $(2,12)$ and $(0,4)$.

- a. -2
- b. 4
- c. $\sqrt{3}$
- d. 3
- e. -4

$$19. \frac{3}{3x^2 - 12y^2} - \frac{5}{6x - 12y} =$$

- a. $\frac{1 - 5x - 10y}{(x - 2y)(x + 2y)}$
- b. $\frac{6 - 5x - 10y}{6(x - 2y)(x + 2y)}$
- c. $\frac{1 - 5x - 57}{(x - 2y)}(x + y)$
- d. $\frac{6 - 5x + 10y}{6(x - 2y)(x + 2y)}$
- e. $\frac{-2}{3x^2 - 6x - 12y^2 + 12y}$

20. A car radiator has a capacity of 17 quarts of fluid. It is full and contains a 41% solution of coolant. Approximately how much of the solution must be drained and replaced by 100% coolant to bring the mixture to a 50 % solution?

- a. 2.6 qt.
- b. 2.125 gal.
- c. 0.8 qt.
- d. 3.2 qt.
- e. 4 qt.

21. Which of the following is an equation of the circle with center at (3,7) and passing through the point (6,3)?

- a. $x^2 - 12x + y^2 - 6y + 20 = 0$
- b. $x^2 - 18y + y^2 + 2y + 57 = 0$
- c. $x^2 - 4x + y^2 - 10y + 9 = 0$
- d. $x^2 - 8x + y^2 - 2y + 9 = 0$
- e. $x^2 - 6x + y^2 - 14y + 33 = 0$

22. If $\left(\frac{1}{2}\right)^{2x-4} = 64$, then 3^{2x+5} is equal to

- a. 9
- b. 3
- c. 27
- d. $\frac{1}{3}$
- e. $\frac{1}{9}$

23. The number of real solutions of $|5x - 6| = e^{2\ln(x)}$ is

- a. 0
- b. 1
- c. 2
- d. 3
- e. 4

24. The graph of the function $f(x) = x^2\sqrt{1-x^2}$ is symmetric with respect to the

- a. x-axis
- b. y-axis
- c. origin
- d. line $y=1$
- e. line $y=x$

25. Given the matrix equation $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \cdot A = \begin{bmatrix} 7 \\ 14 \end{bmatrix}$, A equals

- a. $[3, 2]$
- b. $[0, 3.5]$
- c. $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$
- d. $\begin{bmatrix} 0 \\ 3.5 \end{bmatrix}$
- e. $\begin{bmatrix} 4 \\ 12 \end{bmatrix}$

26. A rectangular garden measuring 80 by 60 meters has its area doubled by adding a border of uniform width along both shorter sides and one longer side. Find the width of the border.

- a. 10
- b. 15
- c. 20
- d. 25
- e. 30

27. The number of complex roots of the equation $(x^2 + x + 1)(2x^2 - x - 1) = 0$ is

- a. 0
- b. 1
- c. 2
- d. 3
- e. 4

28. The horizontal asymptote of $f(x) = \frac{3x^2}{x^2 - 1}$ is

- a. $y = \frac{1}{3}$
- b. $y = 1$
- c. $x = 1$
- d. $x = -1$
- e. $y = 3$

29. Copier A makes x copies in 30 minutes. Copier B makes x copies in 50 minutes. Using both copiers, how many minutes does it take to make x copies?

- a. 19.5
- b. 21.45
- c. 18.75
- d. 16.5
- e. 40

30. Which of the following polynomial equations has roots 1, $3 + 2i$, and $3 - 2i$?

- a. $x^3 - 7x^2 + 19x - 13 = 0$
- b. $x^3 - 8x^2 + 21x - 14 = 0$
- c. $x^3 + 5x^2 - 14x + 8 = 0$
- d. $x^3 + 4x^2 - 15x + 10 = 0$
- e. $x^3 + 6x^2 - 12x + 5 = 0$

31. Find the solution interval for $x^2 - x < 6$.

- a. $(-2, 3)$
- b. $(-4, 10)$
- c. $(-\infty, -4)$
- d. $(6, \infty)$
- e. $(0, 6)$

32. The solution set for the following system is:

$$\begin{aligned} y - 5x &= -6 \\ 16x^2 - 19x - y &= 1 \end{aligned}$$

- a. $\left\{ \left(\frac{5}{4}, \frac{1}{4} \right), \left(\frac{1}{4}, -\frac{19}{4} \right) \right\}$
- b. $\{(2, 4)\}$
- c. $\{(2, 4), (1, -4)\}$
- d. $\{(4, 25)\}$
- e. $\left\{ \left(\frac{5}{4}, -\frac{1}{4} \right), (1, -4) \right\}$

33. Solve $\log_3(x^2 - 16) = 2$. What is the sum of the solutions?

- a. -2
- b. 0
- c. 3
- d. 4
- e. 6

34. One value of k in the equation $2x^2 - 3kx + 5k^2 = 0$ that makes the product of the roots equal to twice the sum of the roots is

- a. $\frac{6}{5}$
- b. $\frac{5}{6}$
- c. $-\frac{6}{5}$
- d. $-\frac{5}{6}$
- e. $\frac{5}{2}$

35. You run a trail at 7 mph and walk back at 3 mph. If your total time is 2 hours, your round-trip distance is how many miles?

- a. 8.0
- b. 9.2
- c. 8.4
- d. 10.0
- e. 10.2

36. Eight homework scores were 6, 2, 7, 4, 6, 7, 9, 7. The mode and median, respectively, are:

- a. 7,6
- b. 7,6.5
- c. 7,5
- d. 6,7.5
- e. 6,5

37. Find all values of m such that $m^2x^2 + 6x + 3 = 0$ has non-real complex roots.

- a. $0 < m < \sqrt{3}$
- b. $-\sqrt{3} < m < \sqrt{3}$
- c. $m < -\sqrt{3}$
- d. $m > \sqrt{3}$
- e. $|m| > \sqrt{3}$

38. Find all solutions of $\sqrt{31-9x} = 5-x$. What is the sum of the solutions?
- a. 0
 - b. 1
 - c. 3
 - d. 4
 - e. All solutions are extraneous
39. You are playing a video game where the left edge of the screen is the positive y-axis and the bottom of the screen is the positive x-axis. The corners of the screen are at the points (0,0), (0,10), (10,0), and (10,10). Your spaceship is located at the point (3,8) and you shoot toward a target located at the point (1,4). The target moves and your shot hits the left edge of the screen, i.e. the y-axis. Where does your shot hit the y-axis?
- a. (2,0)
 - b. (0,0)
 - c. (0,2)
 - d. (0,3)
 - e. (3,0)
40. Among the zeros of a polynomial with rational coefficients are: -1 , 3 , $1 + \sqrt{5}$, and $3 - 2i$. The least degree the polynomial can have is
- a. 2
 - b. 4
 - c. 6
 - d. 8
 - e. 10