

FORTY-FIFTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 2001

Prepared by:

Mathematics Department
Jackson State CC
Jackson, TN

Reviewed by:

Mathematics Faculty
Austin Peay State University
Clarksville, TN 37044

Coordinated by: Yvette Spore/Tim Britt

Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

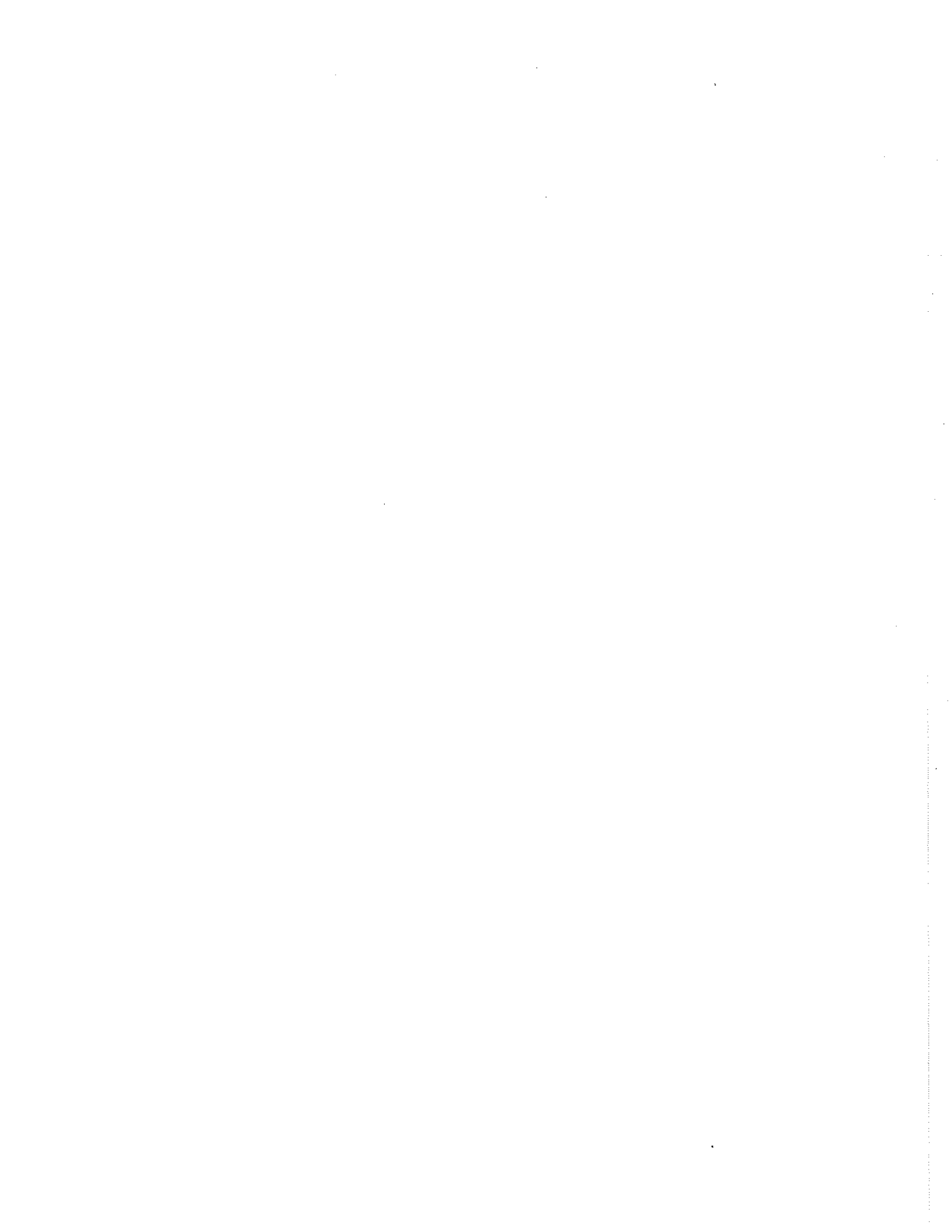
This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, Tennessee
Donnelley Printing Company, Gallatin, Tennessee
TRW Commercial Steering Division, Lebanon, Tennessee
Wright Industries, Inc., Nashville, Tennessee



1. Write $\ln(2x + 1) = y$ in exponential form.
- a. $10^y = 2x + 1$ b. $10^{2x+1} = y$ c. $e^{2x+1} = y$ d. $x = \frac{e^y + 1}{2}$ e. $e^y = 2x + 1$
2. Calculate i^{342} .
- a. i b. 1 c. -1 d. $-i$ e. $2i$
3. Divide:
- $$\frac{2.8 \times 10^5}{4 \times 10^{-3}}$$
- a. 2.4×10^8 b. 7×10^8 c. 7×10^{-16} d. 7×10^7 e. 70
4. Find the x-intercepts of the parabola given by $f(x) = -3x^2 - 10x + 8$.
- a. $-\frac{2}{3}$ and 4 b. $\frac{4}{3}$ and 2 c. $\frac{2}{3}$ and -4 d. $-\frac{4}{3}$ and -2 e. $\frac{3}{2}$ and -4
5. Suppose twenty identical tokens are numbered consecutively from 1 through 20 and placed in an urn. If a token is drawn at random from the urn, what is the probability that the number on the token is a prime number?
- a. $\frac{1}{20}$ b. $\frac{2}{5}$ c. $\frac{9}{20}$ d. $\frac{1}{8}$ e. $\frac{7}{20}$
6. Solve $\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$ for a.
- a. $a = \frac{bf}{f-b}$ b. $a = f - b$ c. $a = \frac{bf}{b-f}$ d. $a = \frac{f-b}{bf}$ e. $a = \frac{1}{f} - \frac{1}{b}$
7. State the domain of the function $f(x) = (4x + 12)^{1/2}$ in interval notation.
- a. $[-3, \infty)$ b. $(-3, \infty)$ c. $(-\infty, -3]$ d. $\{x \mid x \text{ is a real number}\}$ e. $[-3, 3]$
8. Determine the number of real zeros of $y = (x + 4)(x^2 - 14x + 36)$
- a. none b. one c. two d. three e. four
9. Find the equation of the line containing the point $(-1, 5)$ and perpendicular to the line $3x + y = -2$.
- a. $y = -\frac{1}{3}x + \frac{14}{3}$ b. $y = \frac{1}{3}x + \frac{16}{3}$ c. $y = 3x + 8$ d. $y = -3x + 2$
- e. $y = \frac{1}{3}x - \frac{14}{3}$

10. If $g(x) = 6x - 5$, find $\frac{g(-2+h) - g(-2)}{h}$.
- a. $\frac{6h-10}{h}$ b. $\frac{6h+5}{h}$ c. $\frac{6h-5}{h}$ d. $\frac{6h-34}{h}$ e. 6

11. Solve:
- $$\begin{aligned} x + 2y - z &= 8 \\ 2x + y + z &= 1 \\ 3x - y + 2z &= -5 \end{aligned}$$
- If $(1, a, b)$ is a solution, then $a + b =$

- a. -2 b. -1 c. 0 d. 1 e. 2

12. Evaluate the determinant:

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & 2 \\ -1 & 0 & 3 \end{vmatrix}$$

- a. -12 b. -4 c. 12 d. 18 e. 20

13. A rowing team rowing with the current traveled 45 miles in 3 hours. Against the current, the team rowed 27 miles in 3 hours. Find the rate of the rowing team in calm water and the rate of the current.

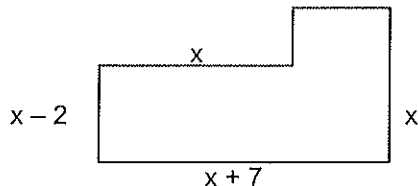
- a. team: 15 mph; current: 6 mph e. Cannot be determined from the given information.
 b. team: 15 mph; current: 3 mph
 c. team: 12 mph; current: 2 mph
 d. team: 12 mph; current: 3 mph

14. Simplify:

$$\left(\frac{-8a^3b^2}{12a^2b^3} \right)^{-3}$$

- a. $-\frac{b^3}{216a^3}$ b. $-\frac{3b^3}{2a^3}$ c. $\frac{27b^3}{8a^3}$ d. $-\frac{27b^3}{8a^3}$ e. $\frac{-64b^3}{a^3}$

15. Find the area of the figure shown below. All dimensions given are in meters.



- a. $(x^2 - 2x + 7)m^2$ b. $(x^2 + 5x)m^2$ c. $(x^2 + 3x - 14)m^2$ d. $(x^2 - 9x)m^2$
 e. $(x^2 + 5x - 14)m^2$

16. The remainder when $2x^4 - 5x^3 - 8x^2 - 17x + 4$ is divided by $x - 4$ is:

- a. 0 b. -384 c. 8 d. -8 e. 136

17. The length of a rectangle is 7 cm and the width is 4 cm. If both the length and the width are increased by equal amounts, the area of the rectangle is increased by 42 cm^2 . Find the amount of the increase in each dimension.

- a. 14 cm b. 3 cm c. 6 cm d. 7 cm e. 4 cm

18. Rewrite $(x^3y^2)^{3/5}$ as a radical expression.

- a. $\sqrt[3]{x^9y^6}$ b. $\sqrt[5]{x^9y^6}$ c. $\sqrt[5]{x^{15}y^{10}}$ d. $\sqrt[3]{x^{15}y^{10}}$ e. $\sqrt[5]{x^6y^5}$

19. Divide:

$$\frac{1-4i}{4+i}$$

- a. $\frac{8}{17} - i$ b. $-\frac{17}{15}i$ c. $-\frac{16}{15}i$ d. i e. $-i$

20. Solve: $\sqrt{3x-2} = 2\sqrt{x} - 1$ The sum of the solution(s), if any, is

- a. 1 b. 9 c. 10 d. -10 e. no real solutions exist

21. Simplify:

$$\frac{x}{x+4} - \frac{4-x}{x^2-16}$$

- a. $\frac{x+1}{x+4}$ b. $\frac{2x-4}{(x+4)(x-4)}$ c. $\frac{-4}{x^2-16}$ d. $\frac{x-4}{x+4}$ e. $\frac{x^2-5x-4}{x^2-16}$

22. Simplify:

$$\frac{\frac{x}{x+4} - \frac{x}{x-4}}{\frac{x}{x+4} + \frac{x}{x-4}}$$

- a. $-\frac{4}{x}$ b. 0 c. 1 d. $\frac{x-4}{x+4}$ e. $-8x$

23. A chemist has some 8% hydrogen peroxide solution and some 3% hydrogen peroxide solution. How many milliliters of the 8% solution should be used to make a 500 ml solution that is 4.2% hydrogen peroxide?

- a. 100 ml b. 80 ml c. 120 ml d. 140 ml e. 380 ml

24. The current (I) in a wire varies directly as the voltage (v) and inversely as the resistance (r). If the current is 32 amps when the voltage is 192 volts and the resistance is 6 ohms, find the current when the voltage is 210 volts and the resistance is 12 ohms.

- a. 2520 amps b. 1260 amps c. 222 amps d. 35 amps e. 17.5 amps

25. Solve: $x^{2/3} - x^{1/3} - 2 = 0$ The sum of the solutions is:

- a. 7 b. -7 c. 9 d. -9 e. 1

26. An old pump requires 6 hours longer to empty a pool than does a new pump. With both pumps working, the pool can be emptied in 4 hours. Find the time required for the new pump, working alone, to empty the pool.

- a. $\frac{23}{8}$ hr b. 4 hr c. 6 hr d. $\frac{25}{8}$ hr e. $\frac{12}{5}$ hr

27. Solve: $\frac{(x+7)(x-4)}{x+10} \geq 0$

- a. $\{x \mid -10 \leq x \leq -7 \text{ or } x \geq 4\}$ b. $\{x \mid -10 < x \leq -7 \text{ or } x \geq 4\}$ c. $\{x \mid -10 < x < -7 \text{ or } x > 4\}$
 d. $\{x \mid x \leq -10 \text{ or } -7 \leq x \leq 4\}$ e. $\{x \mid x < -10 \text{ or } -7 \leq x \leq 4\}$

28. A pool is treated with a chemical to reduce the amount of algae. The amount of algae in the pool t days after the treatment can be approximated by the function $A(t) = 30t^2 - 300t + 4000$. How many days after treatment will the pool have the least amount of algae?

- a. 50 days b. 100 days c. 10 days d. 5 days e. 8.4 days

29. If $f(x) = x^2 - 6$ and $g(x) = x + 8$, find $f \circ g$.

- a. $x^2 + 58$ b. $x^2 + 8x + 10$ c. $x^2 + 2$ d. $x^2 + 16x + 58$ e. $x^3 + 8x^2 - 6x - 48$

30. Which pair of functions are inverses of each other?

A: $f(x) = x + 3; g(x) = x - 3$

B: $f(x) = x + 11; g(x) = 11 - x$

- a. neither b. B only c. A and B d. A only e. More information is needed

31. Give all solutions of: $\log_2 24 - \log_2(x^2 - 1) = \log_2 3$

- a. 3 b. $\frac{3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}$ c. 3, -3 d. -3 e. no solution

32. Find a_{16} in the arithmetic sequence $-4, 1, 6, \dots$

- a. 80 b. 79 c. 76 d. 71 e. 66

33. You start a chain letter and send it to three friends. Each of the three friends sends it to three others, and so on. How many letters will have been mailed from the first through the fifth mailings?

- a. 120 letters b. 363 letters c. 1089 letters d. 1092 letters e. 243 letters

34. Find the 4th term of the expansion of $(2x + 1)^6$.

- a. $20x^3$ b. $8x^3$ c. $160x^3$ d. $240x^4$ e. $320x^4$

35. Find the center and radius of the circle given by: $x^2 + y^2 + 6x - 2y + 6 = 0$

- a. (-3, 1); 4 b. (-3, -1); 4 c. (3, -1); 2 d. (-3, -1); 2 e. (-3, 1); 2

36. The graph of $\frac{x^2}{4} - \frac{y^2}{9} = 1$ is a(n):

- a. parabola b. hyperbola c. circle d. ellipse e. pair of intersecting lines

37. Give all solutions of:

$$x^2 + y^2 = 13$$

$$x^2 - y^2 = 5$$

- a. (2, 3), (-2, 3), (-2, -3), and (2, -3) b. (3, 2)
c. (3, 2), (3, -2), (-3, 2), and (-3, -2) d. (3, 2) and (3, -2) e. (3, 2) and (-3, -2)

38. A couple intends to have four children. Assume that having a boy or a girl is an equally likely event. Find the probability that the couple has 4 children of the same sex.

- a. $\frac{1}{2}$ b. $\frac{1}{8}$ c. $\frac{1}{16}$ d. $\frac{1}{3}$ e. $\frac{1}{4}$

39. In a right triangle ABC, $A = 45^\circ$ and side $a = 20$ in. Find c .

- a. $\sqrt{20}$ in. b. $2\sqrt{20}$ in. c. $20\sqrt{2}$ in. d. 40 in. e. $10\sqrt{3}$ in.

40. If $4^{x^2} 2^{-4x} = 8$, then the possible values of x are:

- a. $\frac{2 \pm \sqrt{10}}{2}$ b. $\frac{2 \pm i\sqrt{2}}{2}$ c. $2 \pm \sqrt{3}$ d. $\frac{2 \pm \sqrt{7}}{2}$ e. 3 and 1

