Math 3130 Test 1 100 points

Name:_____ Chapter 7 Spring 2002 Score:

Show all steps in each problem to earn full credit.

1 (a) Show that $L\{t^n\} = \frac{n}{s} L\{t^{n-1}\}$. L denotes the Laplace transform.

(b) Use the formula in part (a) to write the Laplcae transform for n = 1, 2 and 3.

(c) Use the definition of the Laplace transform to f (t), where

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}.$$

(d) Fill in the blanks or answer true or false.

i. If f is not piecewise continuous on $[0,\infty]$, then the Laplace transform of f (t) will not exists.

ii. The function $f(t)=\left(e^{t}\right)^{10}$ is not of exponential order.

iii. $F(s)=\dfrac{s}{s^{2}+4}$ is not the Laplace transform of a function that is piecewise continuous and of exponential order.

iv. The Laplace transform of e^{-5t} exists for s > _____

v. $f(t)=e^{t^{2}}$ is of exponential order.

2. Find the following.

(a) $L^{-1}\left\{\dfrac{s}{(s-2)(s-3)(s-6)}\right\}$

(b) $L\left\{e^{3t}\left(9-4t+10\sin\dfrac{t}{2}\right)\right\}$

(c) $L^{-1}\left\{\dfrac{s}{s^2+4s+5}\right\}$

(d) $L\{\cos 2t\mu(t-\pi)\}$

(e) $L^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+1}\right\}$

(f) $L\{\delta(t-\frac{\pi}{2}) + \delta(t-\frac{3\pi}{2})\}$

3. Use the formula $\int_0^t f(\tau)d\tau = L^{-1}\left\{\dfrac{F(s)}{s}\right\}$ to evaluate $L^{-1}\left\{\dfrac{1}{s^2(s-1)}\right\}$.

4. Write the given function in terms of the unit step function and find its Laplace transform.

$$f(x)=\begin{cases}2, & 0\le t<3\\ -2, & t\ge 3\end{cases}$$

5. Use the Laplace transform to solve the given system of differential equations.

$$\frac{dx}{dt}=-x+y$$
$$\frac{dy}{dt}=2x$$
$$x(0)=0,\ y(0)=1$$