

Name:

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1. Let

$$\mathbf{X}_1 = \begin{pmatrix} 6 \\ -1 \\ -5 \end{pmatrix} e^{-t}, \quad \mathbf{X}_2 = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} e^{-2t}, \quad \mathbf{X}_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} e^{3t}$$

(a) Show that $\mathbf{X}_1, \mathbf{X}_2$ and \mathbf{X}_3 form a fundamental set of solutions on $(-\infty, \infty)$.

(b) Show that the vectors $\mathbf{X}_1, \mathbf{X}_2$ and \mathbf{X}_3 are solutions of the system

$$\mathbf{X}' = \begin{pmatrix} 0 & 6 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \mathbf{X}$$

(c) Prove that by the superposition principle the linear combination

$$\mathbf{X} = c_1 \begin{pmatrix} 6 \\ -1 \\ -5 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} e^{3t}$$

is the general solution of the system in part (b).

2. Find the general solution of the system

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \mathbf{X}$$

3. Use variation of parameters to solve the system

$$\mathbf{X}' = \begin{pmatrix} 3 & -5 \\ \frac{3}{4} & -1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{\frac{t}{2}}$$

4. Use the Laplace transform to find the general solution of the system

$$\mathbf{X}' = \begin{pmatrix} 4 & 3 \\ -4 & -4 \end{pmatrix} \mathbf{X}$$