

Math 3120 Final Examination

Name..... 100 Points Fall 2001 core.....

Show all steps in each problem to earn full credit.

1. a. Use the substitution $y = e^{mx}$ to find the roots of the auxiliary equation of the second-order differential equation $ay'' + by' + cy = 0$.

b. If $m_1 = m_2$, find an expression for y_2 , where $y_2 = y_1(x) \int \frac{e^{-\int P(x) dx}}{y_1^2(x)} dx$.

- c. Solve $x^2y'' - xy' + 2y = 0$ where $y_1 = x \sin(\ln x)$ using the formula in part (b).

d. Find the general solution of $\frac{d^5y}{dx^5} + 5\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} - 10\frac{d^2y}{dx^2} + \frac{dy}{dx} + 5y = 0$.

2. Use the annihilator approach to solve the initial-value problem
 $y'' + y = 8 \cos 2x - 4 \sin x$, $y(\frac{\pi}{2}) = -1$, $y'(\frac{\pi}{2}) = 0$.

3. Solve the given system of differential equations by systematic elimination.

$$\frac{dx}{dt} = -y + t$$

$$\frac{dy}{dt} = x - t$$

4a. Determine the singular points of each differential equation and classify them.

i. $x^3y'' + 4x^2y' + 3y = 0$

ii. $(x^2 - 9)y'' + (x + 3)y' + 2y = 0$

iii. $x(x + 3)^2y'' - y = 0$

iv. $(x^3 + 4x)y'' - 2xy' + 6y = 0$

Let $x = 0$ be a regular singular point of the differential equation $2xy'' - y' + 2y = 0$.

b. Show that the indicial roots of the singularity do not differ by an integer.

c. Use the Frobenius method to obtain two linearly independent series solutions about $x = 0$ and form the general solution on $(0, \infty)$.

5. a. Find the indicial roots of the Bessel's Equation $x^2 y'' + xy' + (x^2 - \nu^2)y = 0$.

b. Find the recurrence relation that generates consecutive coefficients of the assumed solution $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ the given DE.

- c. Using the coefficients $c_{2n} = \frac{(-1)^n}{2^{2n+v} n! \Gamma(1+v+n)}$, the indicial root $r = v$, and the series solution $y = \sum_{n=0}^{\infty} c_{2n} x^{2n+v}$, write the expressions for the two solutions using the notations $J_v(x)$ and $J_{-v}(x)$.

- d. Write the general solution of each DE using the notations in part (c).

i. $x^2 y'' + xy' + \left(x^2 - \frac{1}{9}\right) y = 0$ *ii.* $4x^2 y'' + 4xy' + (4x^2 - 25) y = 0$

