9. A farmer has 160 feet of fencing to enclose 2 adjacent rectangular pig pens. What dimensions should be used so that the enclosed area will be a maximum?

(a) $4\sqrt{15}$ ft by $\frac{8}{5}\sqrt{15}$ ft (b) 40 ft by $\frac{80}{3}$ ft (c) 20 ft by $\frac{80}{3}$ ft

(d) 40 ft by 40 ft (e) None of these

1—M—Answer: c

10. The management of a large store wishes to add a fenced-in rectangular storage yard of 20,000 square feet, using the building as one side of the yard. Find the minimum amount of fencing that must be used to enclose the remaining 3 sides of the yard.

(a) 400 ft (b) 200 ft (c) 20,000 ft

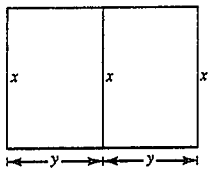
(d) 500 ft (e) None of these

2—M—Answer: a

11. The management of a large store has 1600 feet of fencing to enclose a rectangular storage yard using the building as one side of the yard. If the fencing is used for the remaining 3 sides, find the area of the largest possible yard.

2—O—Answer: 320,000 square feet

12. A person has 400 feet of fencing to enclose two adjacent rectangular regions of the same size. What dimensions should each region be so that the enclosed area will be a maximum?



1—O—Answer: $66\frac{2}{3}$ ft by 50 ft

13. A dog owner has 25 feet of fencing to use for a rectangular dog run. If he puts the run next to the garage, he only needs to use the wire for three sides. What dimensions will yield the maximum area?

1—O—Answer: $x = 6.25$ ft, $y = 12.5$ ft

14. Two equal rectangular lots are to be made by fencing in a rectangular lot and putting a fence across the middle. If each lot is to contain 1875 square feet what size lots require the minimum amount of fence and what is the minimum amount of fence needed?

2—O—Answer: 50 ft by 37.5 ft for a minimum amount of 300 ft of fence

15. A rectangular field is to be fenced in so that the resulting perimeter is 250 meters. Find the dimensions of that field for which the area is maximum.

1—O—Answer: 62.5 meters by 62.5 meters

16. An open box is to be made from a rectangular piece of material by cutting equal squares from each corner and turning up the sides. Find the dimensions of the box of maximum volume if the material has dimensions 6 inches by 6 inches.

2—O—Answer: 4 in. by 4 in. by 1 in.

17. An open box is to be made from a square piece of material, 12 inches on each side, by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made in this manner.

2—O—Answer: $8^2(2) = 128$ cubic inches

18. An open box is to be made from a rectangular piece of cardboard, 7 inches by 3 inches, by cutting equal squares from each corner and turning up the sides.

a. Write the volume, V, as a function of the edge of the square, x, cut from each corner.

b. Use a graphing utility to graph the function, V. Then use the graph of the function to estimate the size of the square that should be cut from each corner and the volume of the largest such box.

2—O—T—Answer: **a.** $V = x(7 - 2x)(3 - 2x)$ **b.** 0.65 in. by 0.65 in.; $V = 6.3$ cubic inches

19. An open box is to be made from a 3-foot by 4-foot rectangular piece of material by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made in this manner.

(a) 4.01 ft^3 (b) 3.92 ft^3 (c) 3.03 ft^3 (d) 2.08 ft^3 (e) None of these

1—M—T—Answer: c

20. An open box is to be made from a 3-foot by 5-foot rectangular piece of material by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made in this manner.

(a) 5.2 ft^3 (b) 4.1 ft^3 (c) 7.5 ft^3 (d) 3.3 ft^3 (e) None of these

1—M—T—Answer: b

21. An open box is constructed from cardboard by cutting out squares of equal size in the corners and then folding up the sides. If the cardboard is 5 inches by 10 inches, determine the volume of the largest box which can be so constructed.

(a) 24.0 (b) 1.1 (c) 14.7 (d) 3.4 (e) None of these

2—M—T—Answer: a

22. An open box is to be constructed from cardboard by cutting out squares of equal size in the corners and then folding up the sides. If the cardboard is 6 inches by 11 inches, determine the volume of the largest box which can be so constructed.

2—O—T—Answer: 37.2 $in.^3$

23. Find the point on the graph of $y = \sqrt{x + 1}$ closest to the point $(3, 0)$.

(a) $(0, 1)$ (b) $\left(\frac{5}{2}, \sqrt{\frac{7}{2}}\right)$ (c) $(3, 2)$

(d) $\left(2, \sqrt{3}\right)$ (e) None of these

2—M—Answer: b

24. Find the point on the graph of $y = x^3$ closest to the point $(2, 0)$. Find the x-value accurate to the nearest 0.1.

(a) $(1.2, 1.728)$ (b) $(0.8, 0.512)$ (c) $(1.8, 5.832)$

(d) $(1, 1)$ (e) None of these

1—M—T—Answer: b

25. Find the point on the graph of $y = x^3$ closest to the point $(1, 0)$. Find the x-value accurate to the nearest 0.1.

1—O—T—Answer: $(0.7, 0.3)$

26. A page is to contain 45 square inches of print. The margins at the top and bottom of the page are each $1\frac{1}{2}$ inches wide. The margins on each side are 1 inch. What should be the dimensions of print so that a minimum amount of paper is used?

2—O—Answer: $\sqrt{30}$ in. by $\dfrac{3\sqrt{30}}{2}$ in.

27. A page is to contain 60 square inches of print. The margins at the top and bottom of the page are each $1\frac{1}{2}$ inches wide. The margins on each side are 1 inch. What should be the dimensions of print so that a minimum amount of paper is used?

(a) 6.3 in. by 9.5 in. (b) 5.9 in. by 10.2 in. (c) 5.8 in. by 10.3 in.

(d) 5.3 in. by 11.3 in. (e) None of these

2—M—T—Answer: a

28. A manufacturer determines that x employees on a certain production line will produce y units per month where $y = 75x^2 - 0.2x^4$. To obtain maximum monthly production, how many employees should be assigned to the production line?

2—O—Answer: 14

29. A right circular cylinder is to be designed to hold 22 cubic inches of a soft drink. The cost for the material for the top and bottom of the can is twice the cost for the material of the sides. Let r represent the radius and h the height of the cylinder.

a. Write the equation for the surface area, SA, in r and h.

b. Write the cost function, C.

c. Write the cost function as a function of one variable, r.

d. Find the radius that minimizes cost.

2—O—Answer: **a.** $SA = 2\pi rh + 2\pi r^2$

b. $C = 2\pi krh + 4\pi kr^2$, k constant

c. $C = \dfrac{44k}{r} + 4\pi kr^2$

d. $r = \sqrt[3]{\dfrac{11}{2\pi}} \approx 1.2$ in.

3.8 Newton's Method

1. Calculate 3 iterations of Newton's Method to approximate the real zero of $f(x) = -x^3 + x + 1$. Use $x_1 = 1.0000$ as the initial guess and round to 4 decimal places after each iteration.

(a) 1.5432 (b) 1.1056 (c) 1.3252
(d) 1.3198 (e) None of these

1—M—Answer: c

2. Calculate 3 iterations of Newton's Method to approximate the real zero of $f(x) = x^3 - 2x - 2$. Use $x_1 = 1.5000$ as the initial guess and round to 4 decimal places after each iteration.

(a) 1.7693 (b) 1.9867 (c) 2.0003
(d) 1.8196 (e) None of these

1—M—Answer: a

3. Calculate 3 iterations of Newton's Method to approximate the real zero of $f(x) = x^3 - x + 1$. Use $x_1 = 1.5000$ as the initial guess and round to 4 decimal places after each iteration.

1—O—Answer: -1.3247

4. Use Newton's Method to approximate the real zero of the function in the interval $[-1, 0]$: $f(x) = x^3 + x + 1$.

(a) -0.83 (b) -0.68 (c) -0.48
(d) -0.23 (e) None of these

2—M—Answer: b

5. Use Newton's Method to approximate the real zero of the function in the interval $[0, 1]$: $f(x) = 3x^3 + x^2 - 16x + 10$.

(a) 0.58 (b) 0.65 (c) 0.73
(d) 0.94 (e) None of these

2—M—Answer: c

6. Use Newton's Method to approximate the real zero of the function in the interval $[0, 1]$: $f(x) = x^3 + 2x - 1$.

(a) 0.450 (b) 0.453 (c) 0.456
(d) 0.459 (e) None of these

2—M—Answer: b

7. Use Newton's Method to approximate the zero of $f(x) = x^3 + 4x + 2$ in the interval $[-1, 0]$. (Use an accuracy of ± 0.001.)

2—O—Answer: 0.473

8. Use Newton's Method to approximate the zero of $f(x) = x^3 + x + 3x$ in the interval $[-2, -1]$. (Use an accuracy of ± 0.001.)

2—O—Answer: -1.213

9. Given $f(x) = x^3 + x - 7$, use Newton's Method to find the iterative formula for x_{n+1}.

(a) $x_{n+1} = x_n - \dfrac{3x_n^2 + 1}{x_n^3 + x_n - 7}$ (b) $x_{n+1} = x_n - \dfrac{x_n^3 + x_n - 7}{3x_n^2 + 1}$

(c) $x_{n+1} = -\dfrac{x_n^3 + x_n - 7}{3x_n^2 + 1}$ (d) $x_{n+1} = -\dfrac{3x_n^2 + 1}{x_n^3 + x_n - 7}$

(e) None of these

1—M—Answer: b

10. Given $f(x) = x^3 + x + 1$, use Newton's Method to find the iterative formula for x_{n+1}.

(a) $x_{n+1} = x_n - \dfrac{x_n^3 + x_n + 1}{3x_n^2 + 1}$ (b) $x_{n+1} = -\dfrac{x_n^3 + x_n + 1}{3x_n^2 + 1}$

(c) $x_{n+1} = x_n - \dfrac{3x_n^2 + 1}{x_n^3 + n_n + 1}$ (d) $x_{n+1} = -\dfrac{3x_n^2 + 1}{x_n^3 + x_n + 1}$

(e) None of these

1—M—Answer: a

11. Given $f(x) = x^3 + x - 3$, use Newton's Method to find the iterative formula for x_{n+1}.

(a) $x_{n+1} = -\dfrac{x_n^3 + x_n - 3}{3x_n^2 + 1}$ (b) $x_{n+1} = -\dfrac{3x_n^2 + 1}{x_n^3 + x_n - 3}$

(c) $x_{n+1} = x_n - \dfrac{3x_n^2 + 1}{x_n^3 + x_n - 3}$ (d) $x_{n+1} = x_n - \dfrac{x_n^3 + x_n - 3}{3x_n^2 + 1}$

(e) None of these

1—M—Answer: d

12. Given $f(x) = x^3 + x - 1$, use Newton's Method to find the iterative formula for x_{n+1}.

(a) $x_{n+1} = -\dfrac{3x_n^2 + 1}{x_n^3 + x_n - 1}$ (b) $x_{n+1} = -\dfrac{x_n^3 + x_n - 1}{3x_n^2 + 1}$

(c) $x_{n+1} = x_n - \dfrac{x_n^3 + x_n - 1}{3x_n^2 + 1}$ (d) $x_{n+1} = x_n - \dfrac{3x_n^2 + 1}{x_n^3 + x_n - 1}$

(e) None of these

1—M—Answer: c

13. Use a graphing utility to graph the function $f(x) = \dfrac{x^2 - 4x + 1}{x^2 + 2}$.

a. Use the graph to estimate (to one decimal place) the real zero of $f(x)$ in the interval $[0, 1]$.

b. Approximate the real zero in the interval $[0, 1]$ using the value found in part **a** and Newton's Method until two successive approximations differ by less than 0.001.

2—O—T—Answer: **a.** 0.3 **b.** 0.268

14. Use a graphing utility to graph the function $f(x) = 0.2x^3 + 3.1x - 1.4$.

a. Use the graph to estimate (to one decimal place) the real zero of f.

b. Approximate the zero using the value found in part **a** and Newton's Method until two successive approximations differ by less than 0.001.

2—O—T—Answer: **a.** 0.5 **b.** 0.4459

15. Use a graphing utility to graph the function $f(x) = -2 \sin + \cos x$.

a. Use the graph to estimate (to one decimal place) the real zero in the interval $\left[-\pi, -\dfrac{\pi}{2}\right]$.

b. Approximate the zero using the value found in part **a** and Newton's Method until two successive approximations differ by less than 0.001.

2—O—T—Answer: **a.** -2.7 **b.** -2.6779

16. Use a graphing utility to graph the functions $f(x) = x^2$ and $g(x) = x^3 - 2x + 5$ on the same coordinate axes.

a. Use the graphs to estimate (to one decimal place) the x-value of the point of intersection.

b. Use Newton's Method to approximate this point of intersection.

2—O—T—Answer: **a.** -1.8 **b.** $(-1.757, 3.087)$

17. Use Newton's Method to approximate the x-value of the point on the graph of $f(x) = \sqrt{x}$ that is closest to the point (2, 3).

(a) 2 (b) 2.4570 (c) 2.4211

(d) 2.6316 (e) None of these

2—M—Answer: b

18. Solve $\sin x = \dfrac{x}{3}$ in the interval $(0, 2\pi]$. Use a graphing utility to obtain an initial guess and then use Newton's Method until two successive approximations differ by less than 0.001.

2—O—T—Answer: **a.** 2.2789

19. Use a graphing utility to graph the functions $f(x) = x^2$ and $g(x) = \sin x$ on the same coordinate axes.

a. Use the graphs to estimate (to one decimal place) the x-value of the point of intersection where $x > 0$.

b. Use Newton's Method to approximate this point of intersection. (Use an accuracy of ± 0.001.)

2—O—T—Answer: **a.** 0.9 **b.** 0.8767

3.9 Differentials

1. The side of a cube is measured to be 3.0 inches. If the measurement is correct to within 0.01 inch, use differentials to estimate the propagated error in the volume of the cube.

(a) ± 0.000001 in.^{3} (b) ± 0.06 in.^{3} (c) ± 0.027 in.^{3}

(d) ± 0.27 in.^{3} (e) None of these

1—M—Answer: d

2. The radius of a sphere is measured to be 3.0 inches. If the measurement is correct to within 0.01 inch, use differentials to estimate the propagated error in the volume of the sphere.

(a) ± 0.000001 in.^{3} (b) $\pm 0.36\pi$ in.^{3} (c) $\pm 0.036\pi$ in.^{3}

(d) ± 0.06 in.^{3} (e) None of these

1—M—Answer: b

3. The volume of a cube is claimed to be 27 cubic inches, correct to within 0.027 in.^{3} Use differentials to estimate the propagated error in the measurement of the side of the cube.

1—O—Answer: ± 0.001 in.

4. Use differentials to approximate $\sqrt{4.9}$.

(a) 2.225 (b) 2.250 (c) 2.214

(d) 2.450 (e) None of these

1—M—Answer: a

5. Use differentials to approximate $\sqrt{3.4}$.

(a) 1.700 (b) 1.844 (c) 1.850

(d) 1.750 (e) None of these

1—M—Answer: c

6. Use differentials to approximate $\sqrt{3.3}$.

(a) 1.825 (b) 1.817 (c) 1.750

(d) 1.650 (e) None of these

1—M—Answer: a

7. Find the values of dy and Δy for $y = x^3 - 2x$ when $x = 2$ and $\Delta x = 0.1$.

1—O—Answer: $dy = 1 \quad \Delta y = 1.061$

8. Consider $f(x) = x^3$.

   a. Find an equation of the tangent line, T, at the point $(2, 8)$.

   b. Graph f and T on the same coordinate axes using a graphing utility.

   c. Use the graphs to estimate $f(2.1)$ and $T(2.1)$.

   d. Calculate the actual values of $f(2.1)$ and $T(2.1)$.

2—O—T—Answer: **a.** $T(x) = 12x - 16$

c. $f(2.1) \approx 9.5,\ T(2.1) \approx 9.4$

d. $f(2.1) = 9.261,\ T(2.1) = 9.2$

9. Calculate Δy and dy for $f(x) = \dfrac{1}{x}$ when $x = 2$ and $\Delta x = -0.01$.

2—O—Answer: $\Delta y = \dfrac{1}{398} \approx 0.00251,\ dy = 0.0025$

10. Find dy for $y = \dfrac{x^2}{2x^2 + 1}$.

1—O—Answer: $\dfrac{2x}{(2x^2 + 1)^2}\,dx$

11. Find dy for $y = \sqrt{1 - 4x^2}$.

1—O—Answer: $-\dfrac{4x}{\sqrt{1 - 4x^2}}\,dx$

12. Find dy for $y = \sec 3x$.

(a) $\sec 3x\,dx$ (b) $3\sec 3x \tan 3x\,dx$ (c) $3\sec^2 3x\,dx$

(d) $3\tan^2 3x\,dx$ (e) None of these

1—M—Answer: b

13. The measurement of the edge of a piece of square floor tile is found to be 12 inches with a possible error of 0.02 inches.

   a. Use differentials to approximate the maximum possible error in the area of the tile.

   b. Use the answer from part **a** to estimate the relative error.

   c. Use the answer from part **b** to estimate the percentage error.

2—O—Answer: **a.** 0.48 square inches **b.** $\frac{1}{300}$ **c.** $\frac{1}{3}\%$

14. The measurement of the circumference of a circle is found to be 54 centimeters. Approximate the percentage error in computing the area of the circle if the possible error in measuring the circumference is 0.6 centimeters. Round your answer to three decimal places.

2—O—Answer: 2.222%