Group Members:

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Radar Measurement of the Rotation Rate of Mercury[1]

This activity makes use of software from CLEA (Contemporary Laboratory Exercises in Astronomy), which was developed by Gettysburg College in PA. In this activity you will use a simulated radio telescope to acquire a pulse spectra. You will then use the Doppler-shift of this spectrum to determine the radial velocity, orbital period, and rotational period of the planet Mercury. This is the same technique that was originally used by astronomers working at the Arecibo radio observatory to determine the rotation rate of Mercury.

Introduction:

Because Mercury is a small planet whose surface features have low contrast, and because it is so close to the sun that it is rarely visible against a dark sky, it is difficult to determine how fast it is rotating merely by looking at it from Earth. In recent years, however, radar techniques have proven most effective in measuring its speed of rotation. The method you will employ here actually has wider application than just measuring the rotation rate of Mercury. It can be used to study the behavior of other planets as well, from cloud-covered Venus to the rings of the major planets, to the smallest asteroids.

The basic idea of this technique is to use a radio telescope to send a short pulse of electromagnetic radiation of known frequency towards Mercury and then record the spectrum of the returning echo. Depending on the relative position of the Earth and Mercury, the pulse will take between 10 minutes and half an hour to travel to Mercury, bounce off, and return.

By the time the pulse has reached Mercury, it has spread out to cover the entire planet. Because the planet's surface is a sphere, the pulse hits different parts of the planet at different times. The pulse first hits the surface at a point on the line that connects the centers of Earth and Mercury (the "sub-radar point"), and a few microseconds later from points further back, towards the edges of the planet.

The frequencies of the returning echoes are different from the frequency of the pulse sent out because the echoes have bounced off the moving surface of Mercury. Any time a source of radiation is moving towards or away from the observer there will be a Doppler shift in the received frequency that is proportional to the velocity along the line of site.

[1] Adapted from *Radar Measurement of the Rotation Rate of Mercury Student Manual*, CLEA Project, Department of Physics, Gettysburg College, Gettysburg, Pennsylvania.

There are two motions of Mercury that we will be able to investigate in this way. First, the shift of the echo from the sub-radar point will allow us to calculate the orbital velocity of the planet. This echo shows up as a single peak whose frequency is shifted away from the original frequency of the pulse. Second, the echoes from the sides of the planets will allow us to calculate the rotational speed of the planet. These echoes will have two additional peaks (one from each side of the planet) that are shifted in a more or less symmetric way around the sub-radar echo.

Procedure:

Part 1: Controlling the Telescope

You will find the program by going to **Start**, then **Programs**, then **CLEA Labs** and finally **Mercury Rotation**. Choose **login** from the *File* menu and enter your names. You are now on the main page. Select **Start** on the main menu. A control panel appears that has only three buttons, as well as displays to show the frequency the telescope is tuned to and the coordinates it is pointed to in the sky. Begin by pressing the **Tracking** button to turn on the sidereal drive so that the telescope will track the planets as the Earth turns.

Next select **Ephemeris** from the main menu; this activates a program that calculates the position of a planet for any date and time. The default is the current local date and time; click **OK** to accept this default. Do not close the ephemeris window in case you need it later. Now return to the main window and click **Set Coordinates**; click **Yes** when the program asks if you want to use the computed values. The telescope will now slew to the new coordinates.

When the telescope move is complete, click on **Send Pulse** to transmit a radar pulse toward Mercury. A **Pulse Sent** message will appear along with the estimated time until reception of the return pulse. This simulation is real-time, so you will have to wait for the pulse to return (the computer will display an animation showing progress of the pulse). While you are waiting for the return echo, calculate d, x, and y for each of the time intervals as described below.

Upon receipt of the echo, a series of five windows showing the echo spectrum at each of five time intervals will appear – 0 μs, 120 μs, 210 μs, 300 μs, and 390 μs. The first is the echo from the sub-radar point and each of the others is from a point successively further back on the surface of Mercury.

Part 2: Data Recording:

For each of your data windows, you must now measure the frequency of each peak to determine the Doppler shift of that signal. To make a measurement, select the window for the echo you wish to measure, then simply double click the mouse button at the point to be measured. For the sub-radar point, you simply measure the central frequency. But for all the other delayed echoes, you must measure the positions of the left and right "shoulders" on the plots (the point at which the intensity just begins to fall off to zero). The left shoulders will be marked with a red arrow, the right ones with a blue arrow. On some of the plots these points are quite obvious and on others they are not.

After you have measured all of the pulse windows, select **Record Measurements** under **Pulse** in the main menu. A data window will appear containing all of the measurements you have made. Now click **Work Sheet** in the main menu. The worksheet that is displayed is the same as the one at the end of this lab report, and you should fill in the data table before you transfer your numbers to the computer.

The computer only records raw data – it does not do the calculations for you! It does, however, check your calculations for you. (Note the notation used to indicate powers of ten in the work sheet entries. Ten to the power is indicated by **E** so that 5.6 x 10^{-8} would be written as 5.6E-08.)

Calculations:

The rotational velocity of Mercury

You will need the following formulas to carry out the steps necessary to determine the rotation rate of Mercury. Since you know the anticipated time delays of the echoes (see above) you can calculate d, x, and y before you have received the echoes. The remaining calculations require data from the echo spectra.

d (in meters) This is the distance the delayed beam has traveled beyond the sub-radar point.

$$d = \tfrac{1}{2} c\Delta t$$

where **c** is the speed of light (3 x 10^8 meters/sec) and **Δt** is the time delay for the particular pulse in seconds. (Note that 1μs = 10^{-6} sec.)

x (in meters) This is the distance parallel to our line of site from the center of Mercury to the point from which the echo comes back.

$$x = R_{merc} - d$$

where R_{merc}=2.42 x 10^6 meters is the radius of Mercury.

y (in meters) This is the distance perpendicular to our line of sight to the extreme outer edge of the region of Mercury from which the echo comes back. It is found by noting that y is one side of a right triangle whose hypotenuse is the radius of Mercury, and whose other side is **x**.

$$y = \sqrt{(R_{merc}^2 - x^2)}$$

Δf_{total} (in hertz, Hz) This is the shift in frequency due to the rotational velocity alone. Since one edge of Mercury is moving toward you and the other is moving away, the difference in frequency shifts from the two edges, **Δf_{right}** and **Δf_{left}** is twice the shift due to rotational velocity.

$$\Delta f_{total} = \tfrac{1}{2}(\Delta f_{right} - \Delta f_{left})$$

$\Delta f_{corrected}$ (in hertz, Hz) This is the shift in frequency corrected for the fact that this is an echo – the shift is twice that produced by a source which is simply emitting at a known frequency.

$$\Delta f_{corrected} = \tfrac{1}{2}\Delta f_{total}$$

v_0 (in meters/sec) This is the component of the rotational velocity of the edge of Mercury along the line of site at the point from which the echo returns. We simply apply the Doppler shift equation.

$$v_0 = c \times \frac{\Delta f_{corrected}}{f}$$

Where, again, **c** is the speed of light in meters per second, and **f** is the unshifted transmitted frequency of the pulse (which can be read from the control panel of the radio telescope, but note that **f** must be in Hertz, not Megahertz. 1 MHz = 10^6 Hz.)

v (in meters/sec) This is the equatorial rotational velocity of the planet Mercury corrected for the fact that the velocity you measure is only the component of the rotational velocity directed along your line of sight, and that the component perpendicular to the line of sight produces no measurable Doppler shift.

$$v = v_0 \times \frac{R_{merc}}{y}$$

P_{rot} (in days) For each of the delayed echoes you can now calculate a rotational period for the planet by dividing the circumference of Mercury by its velocity and dividing the result (which will be in seconds), by the number of seconds in a day.

$$P_{rot}(days) = \frac{2\pi R_{merc}}{86{,}400 \cdot v}$$

The orbital velocity of Mercury

The orbital velocity of Mercury can be calculated from the above equations with the following substitution.

$$\Delta f_{total} = \Delta f_{sub-radar}$$

Final Results

When you have calculated **P_{rot}** for each of the delayed echoes, make sure your answers are reasonable. Then compute an average period of rotation in days for Mercury from your values.

The Rotation Period of Mercury = ____________________ days.

What is the percentage difference between this period and the accepted value of 59 days?

$$\frac{P_{rot} - 59}{59} \times 100\,\% = \underline{\hspace{6cm}}$$

Orbital Velocity of Mercury = ________________________

Sketch the relative positions of the Earth, Sun, and Mercury, along with the direction of Mercury's orbit. Does your answer for the orbital velocity seem reasonable?

Mercury Data Table

Δt (microseconds)	**120**	**210**	**300**	**390**
d (meters)				
x (meters)				
y (meters)				
left echo frequency (Hz)				
right echo frequency (Hz)				
Δf_{total} (Hz)				
$\Delta f_{corrected}$ (Hz)				
v_0 (meters/sec)				
v (meters/sec)				
P_{rot} (days)				

Δf for sub-radar pulse ______________________

$v_{orbital}$ = ________________________