How Democratic Is Our Democracy?

Using Math to Measure Fairness in Politics

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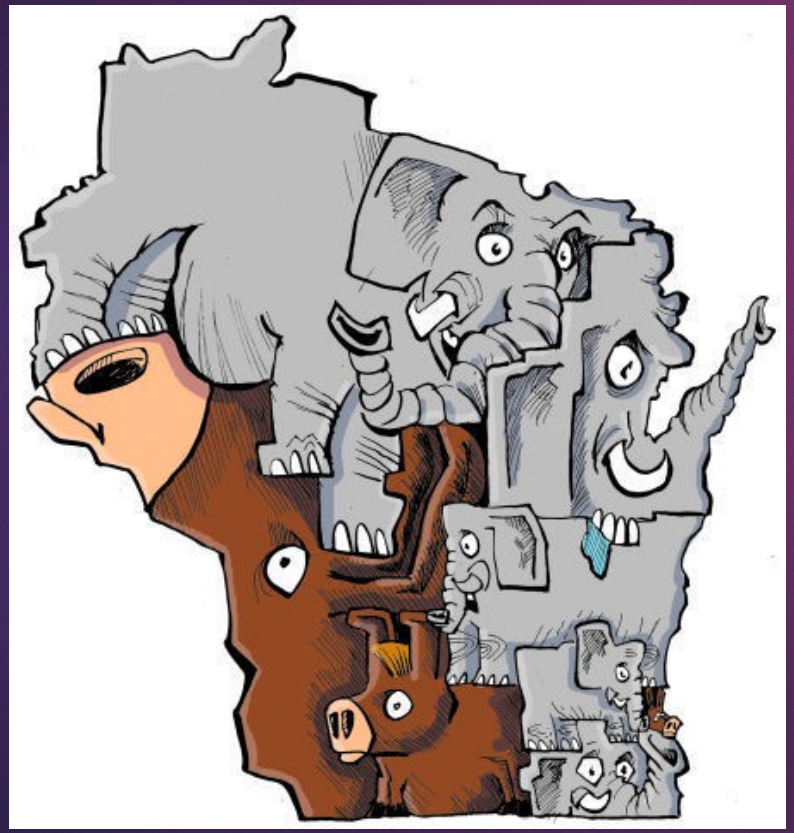
Using Math to Measure Fairness in Politics

- Gerrymandering



- Election methods

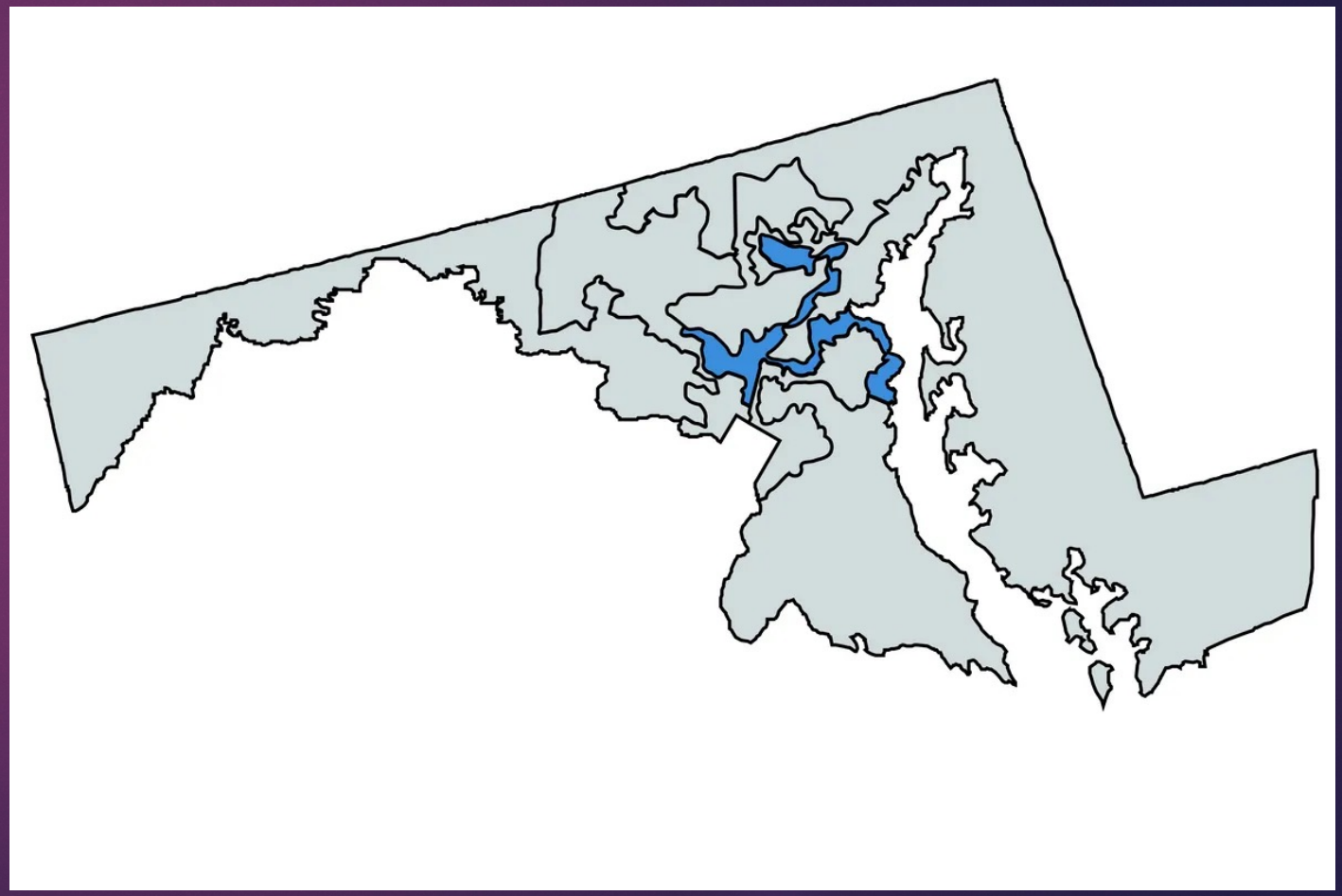
What is Gerrymandering?

- Gerrymandering is the intentional practice of manipulating boundaries through redistricting to gain an advantage to a political party or group.

- Portmanteau for the last name of Governor Elbridge Gerry of Massachusetts and the salamander based on the shape of a new voting district in 1812

Gerrymandering Examples

- Democrats have done it in Maryland
- "broken-winged pterodactyl lying prostrate across the state"
- "blood spatter at a crime scene"



Gerrymandering Examples

- Republicans have done it here in Tennessee

Gerrymandering Examples

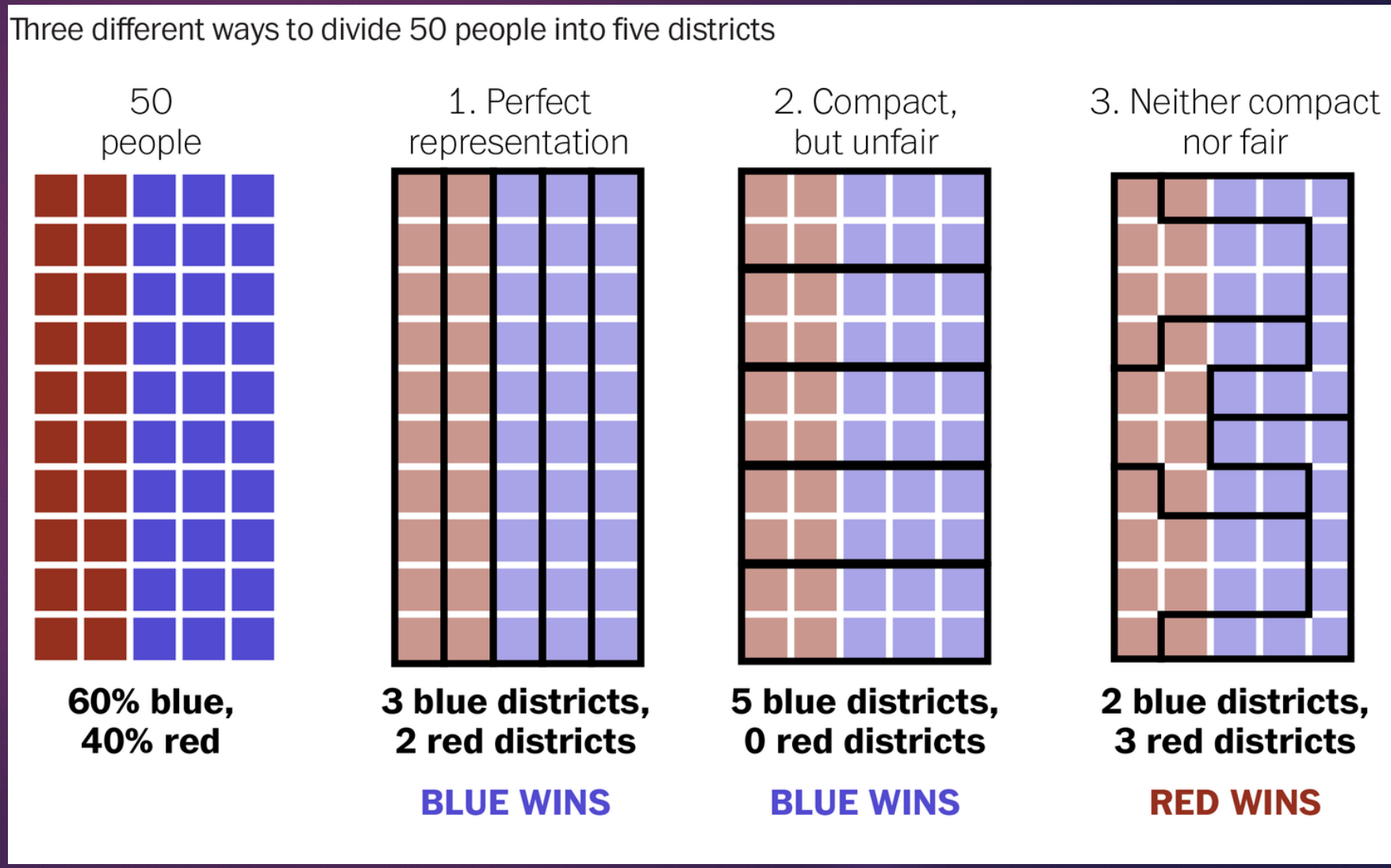
- Republicans have done it here in Tennessee
- The Nashville metropolitan area has been divided into 4 red districts

Why is Gerrymandering Bad?

- Creates a discrepancy between partisan representation in government compared the political leanings of that state
- Can effectively disenfranchises minority groups
- "We have to end the practice of drawing our congressional districts so that politicians can pick their voters and not the other way around." – Barack Obama

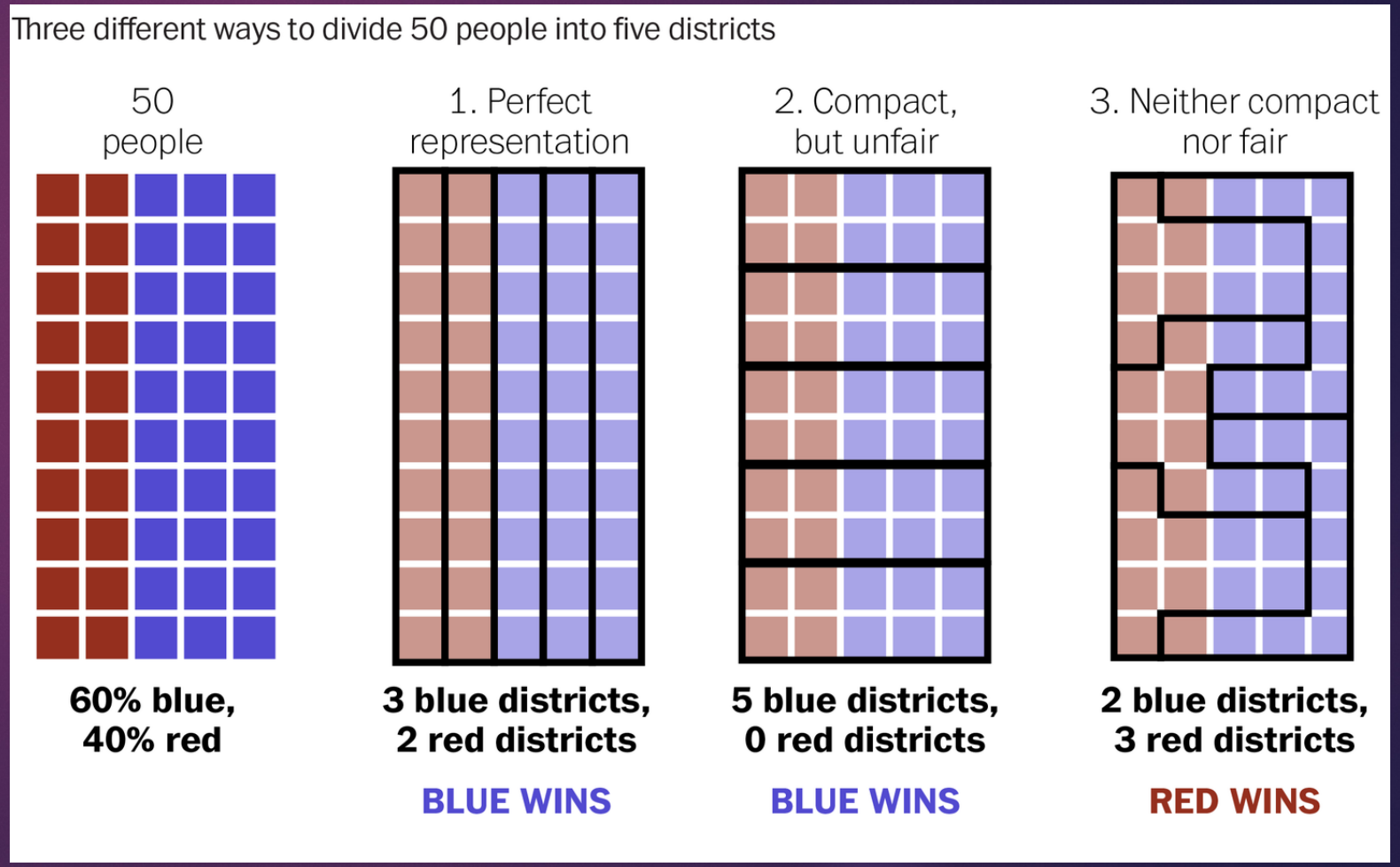
Gerrymandering 101

- Packing – concentrating the opposing party's voting power into one or a few districts
- Cracking – diluting the voting power of the opposing party's supporters across many districts



Using Math to Quantify Gerrymandering

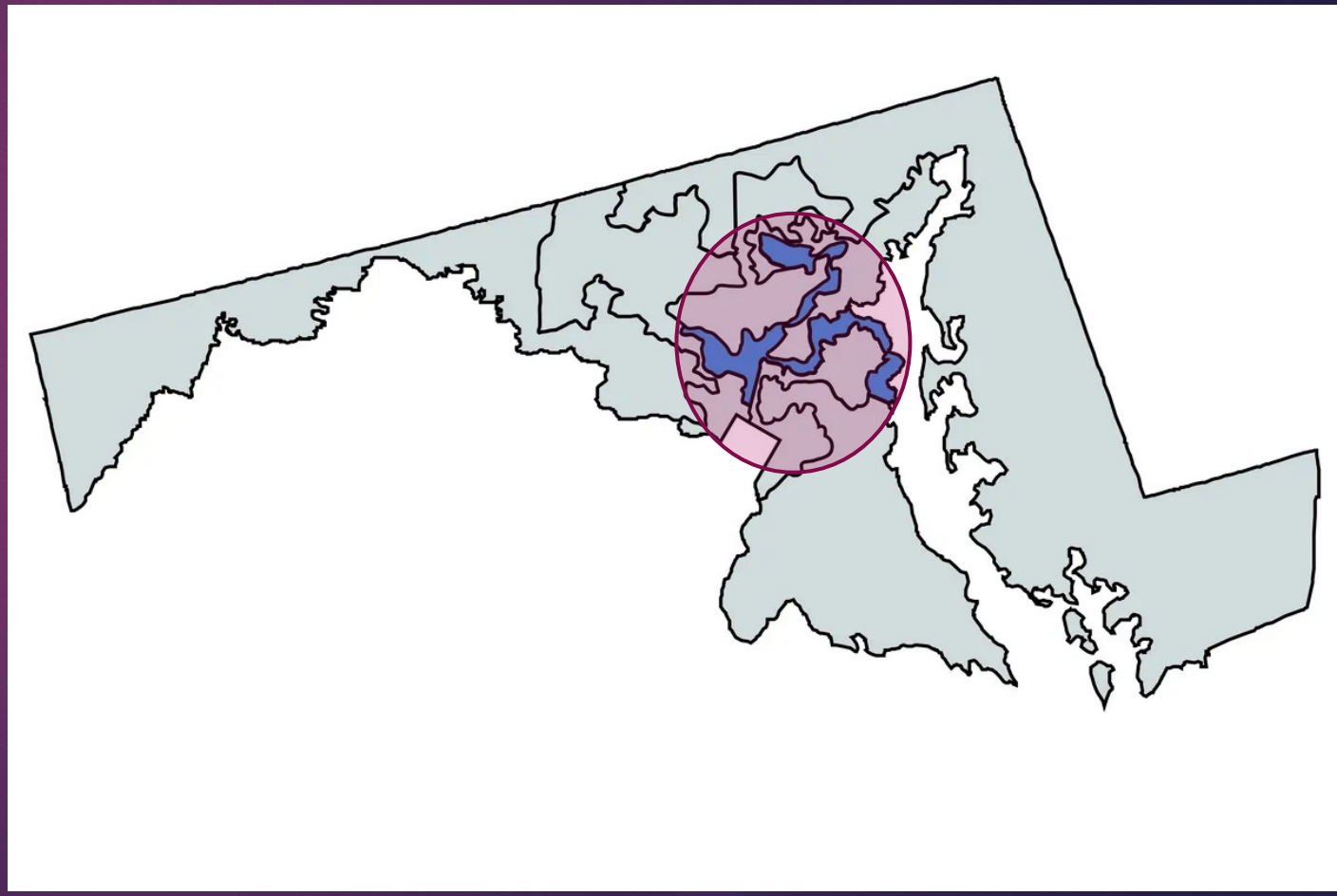
- The Efficiency Gap is a measure to quantify the amount of packing and cracking by calculating the percentage of net wasted votes
- Over 7% is considered gerrymandered
- Based on 2022 votes:
  MD map – 8.1%
  TN map – 10.9%



Using Math to Quantify Gerrymandering

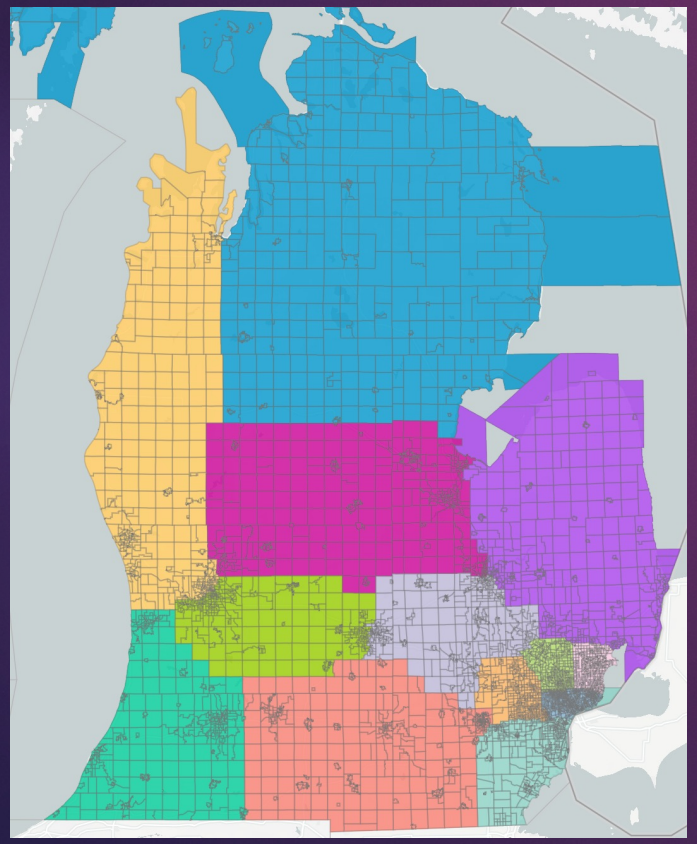
- Ideally, districts should be compact

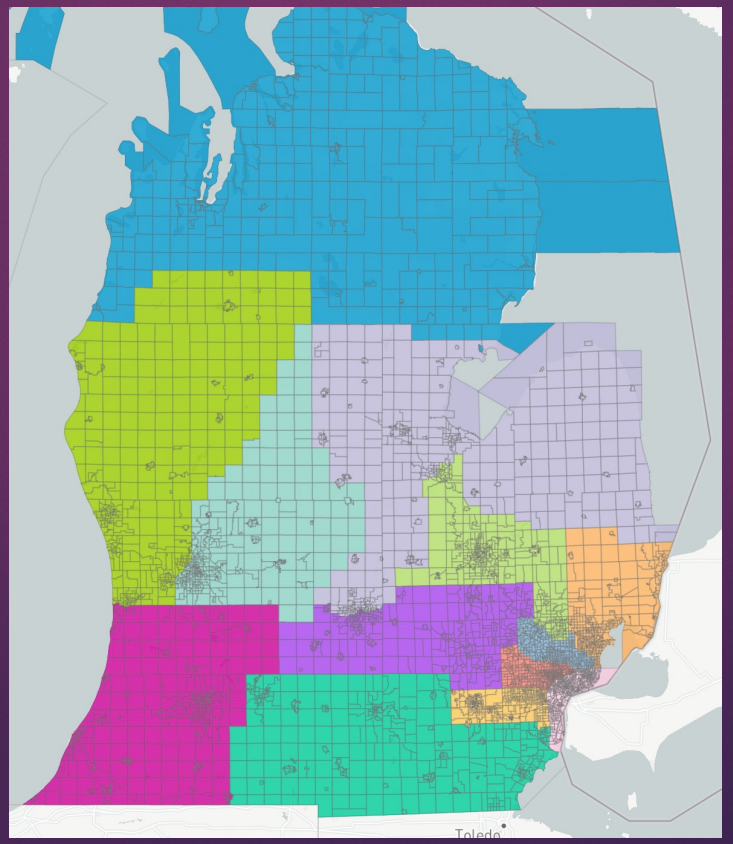
- The Roeck Method:
  Draw the smallest circle that a given district will fit completely within. The Roeck score is the ratio between the area of the district and the area of the circle.

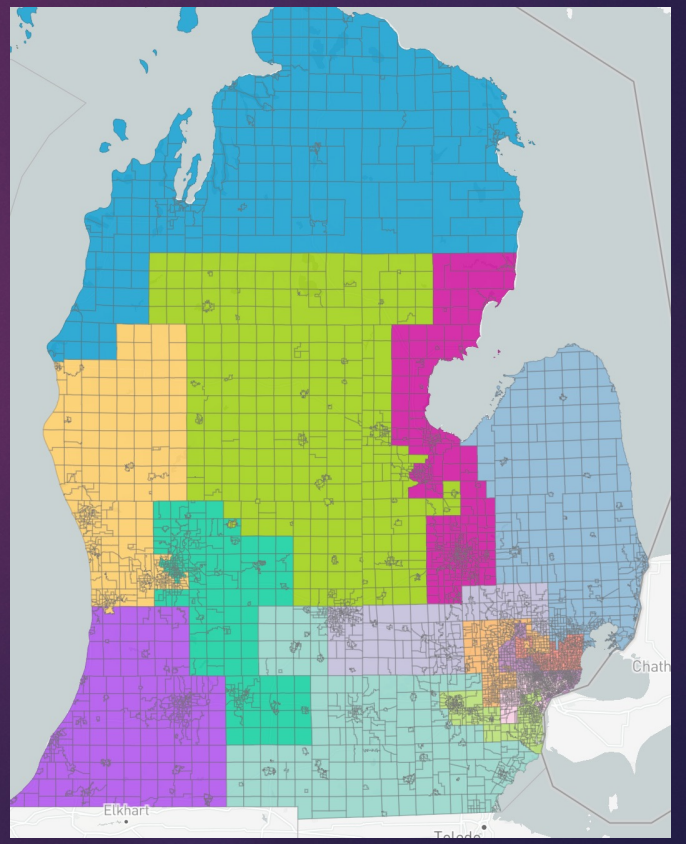


Pop Quiz Time

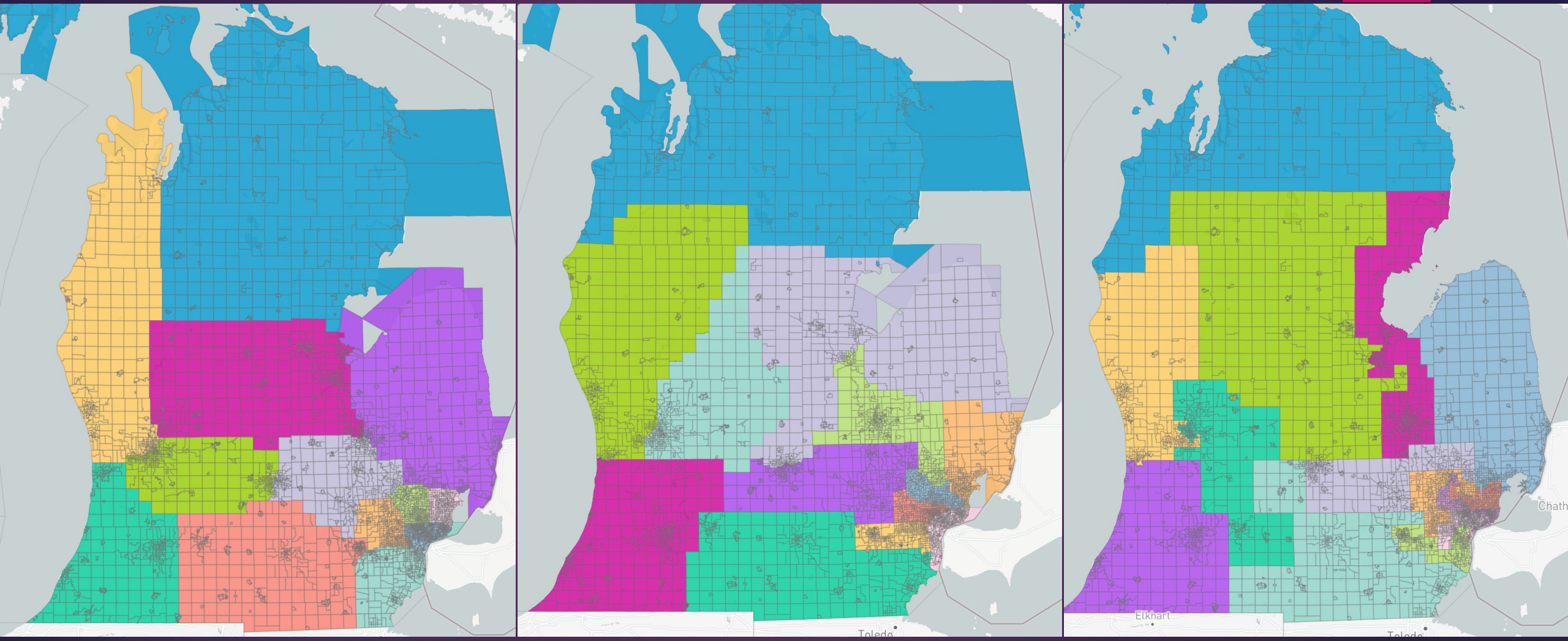
- Which of the following district maps of Michigan are gerrymandered?

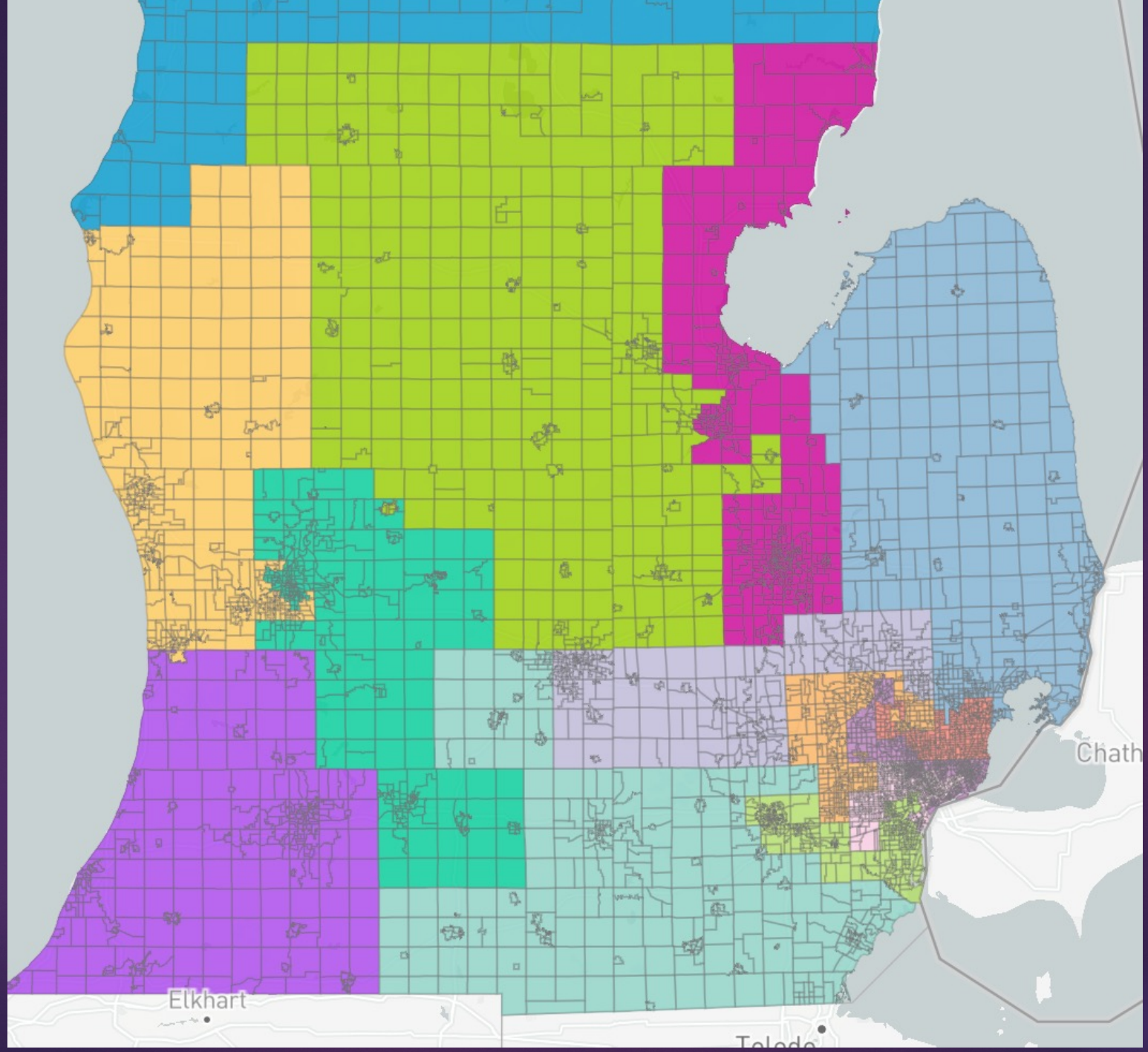






Pop Quiz Time




Elkhart
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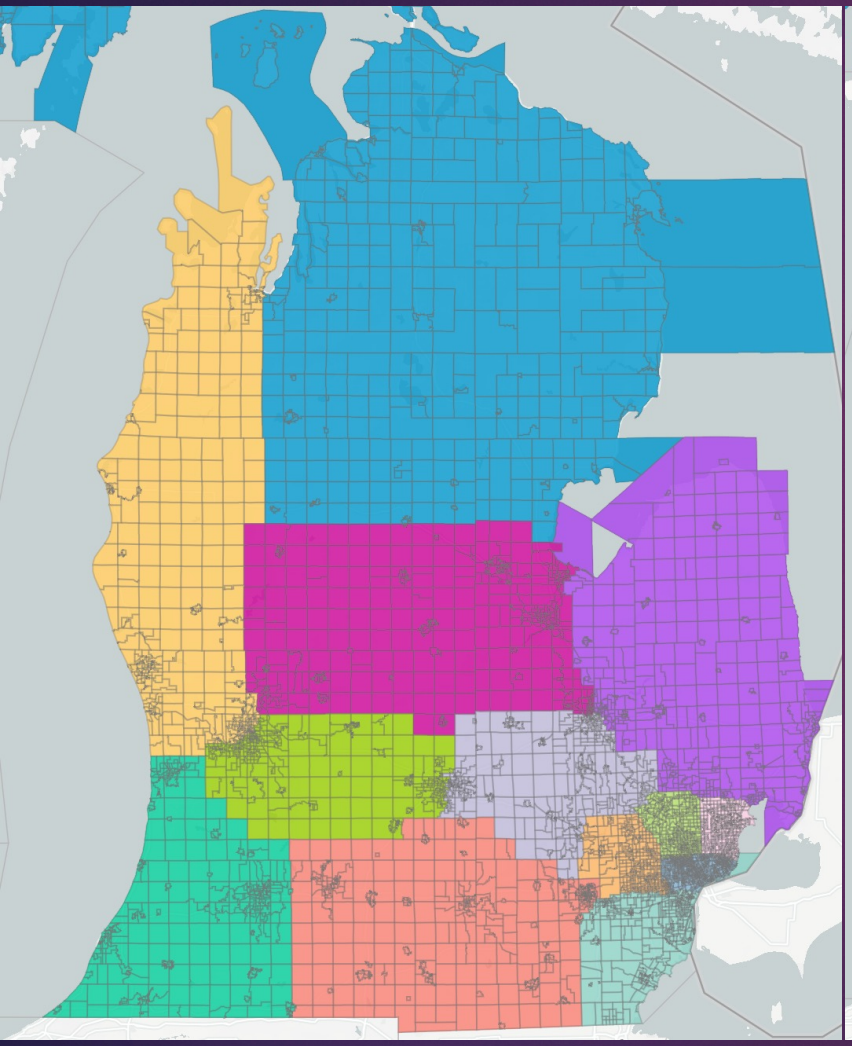
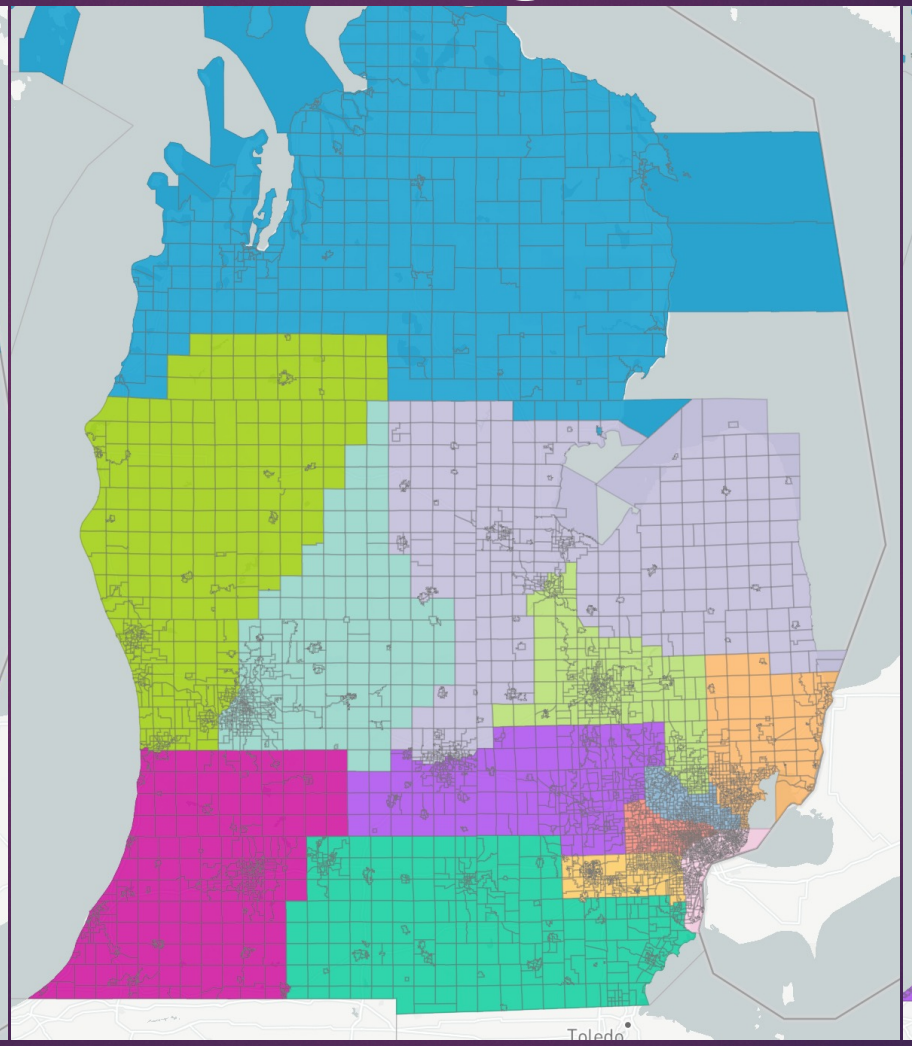
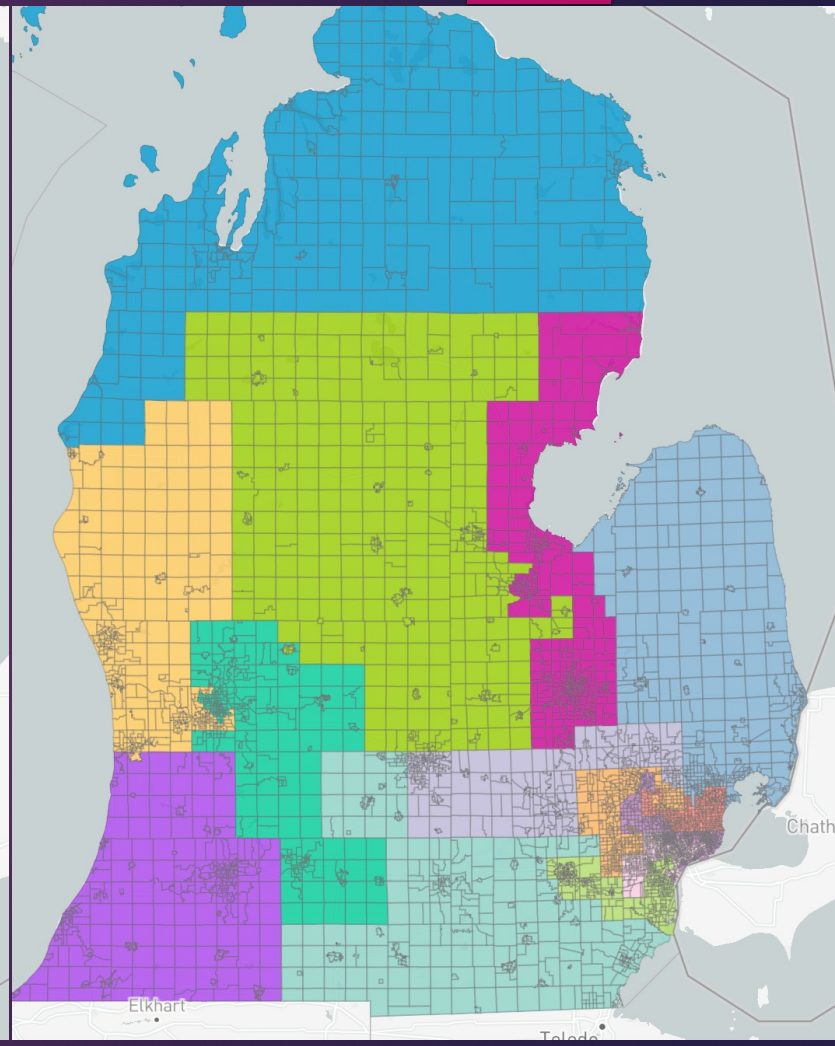
Pop Quiz Time – Michigan 55 D/45 R



11 Dem, 2 Rep



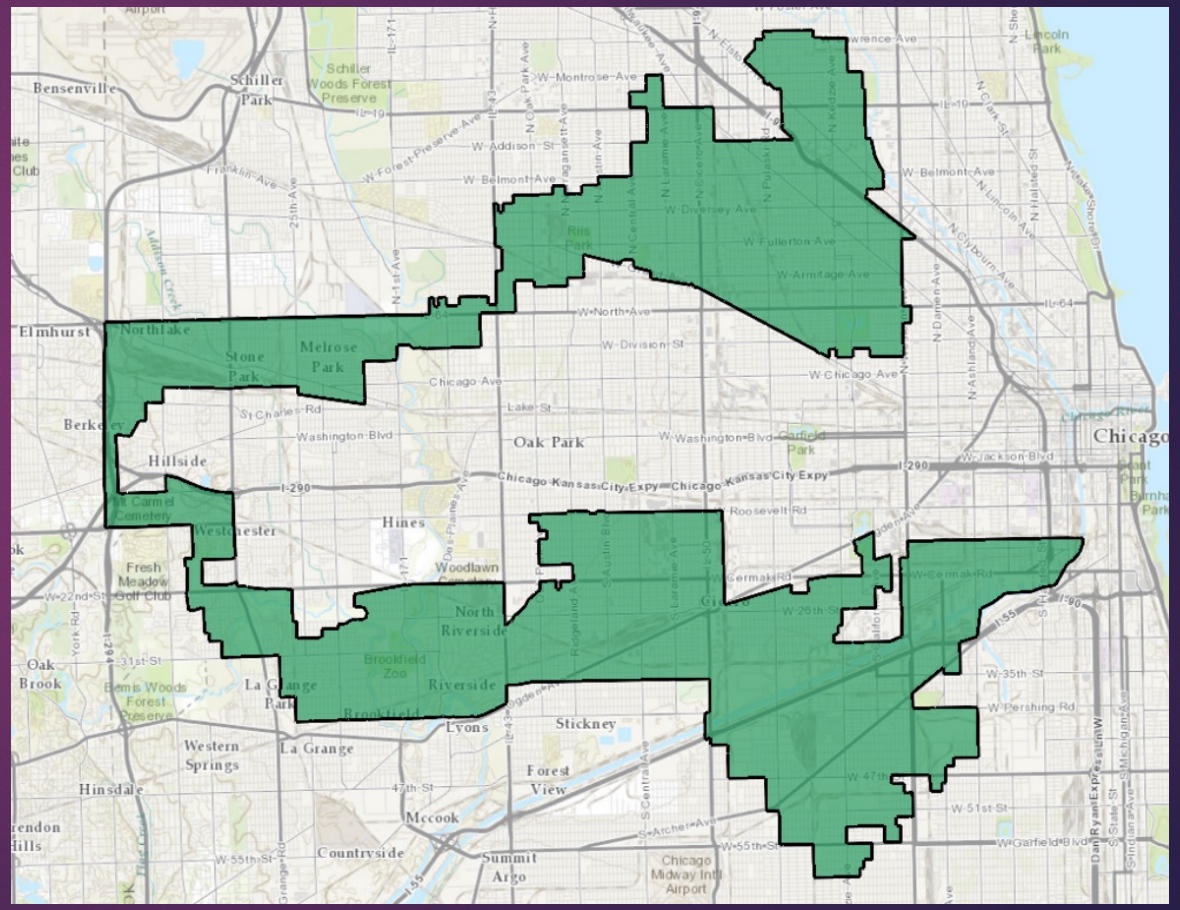
5 Dem, 8 Rep



Actual – 7 Dem, 6 Rep

Can Weird Districts Be Good?

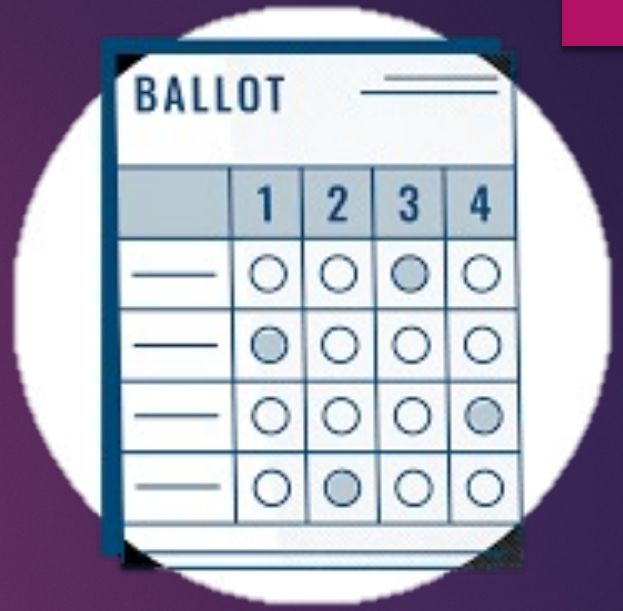
- Illinois' 4th District
- Connects two Hispanic neighborhoods
- Elected the first Latino member of Congress in the Midwest



Voting Methods

- Plurality Method (Popular Vote)
  - Vote for a single candidate, winner receives the most votes

Voting Methods



- Plurality Method (Popular Vote)
  - Vote for a single candidate, winner receives the most votes
- Elimination Method (Ranked Choice Voting)
  - 1. All voters rank every candidate
    - 2. If a candidate has a majority of first-place votes, they win
    - 3. The candidate with the fewest first-place votes is eliminated
    - 4. Redistribute the eliminated candidate's ballots and return to step 2

Ranked Choice Voting in the U.S.

- Two states – Maine and Alaska – use RCV for some state-wide elections
- Alaska will use RCV in the 2024 Presidential election
- 47 cities use RCV for local elections such as NYC, San Francisco, Salt Lake City, and Minneapolis
- Five states including Tennessee have banned the use of RCV in any state or municipal elections

Using Math to Measure Election Fairness

- Fairness Criteria – conditional scenarios where a specific outcome is expected

- Majority Criterion
  - If a candidate receives a majority of first-place votes, then they should win the election
- Unfavorable Majority Criterion
  - If a candidate receives a majority of last-place votes, then they should NOT win the election

Fairness Criteria Violations?

- Clinton 48.18%
- Trump 46.09%
- Johnson 3.28%
- Stein 1.07%
- McMullin .54%
- Sanders .08%

Using Math to Measure Election Fairness

- Condorcet Criterion
  - If a candidate is preferred by voters in pairwise competition over EVERY other candidate, then they should win the election

- Monotonicity Criterion
  - If a candidate would win an election, then after changes in ballots are made that favor that candidate, they should still win the election.

CoSTEM Dean Election

- Four candidates in a Dean election: Karen Meisch (M), Kallina Dunkle (D), Jackie Vogel (V), and Leong Lee (L)
- 80 CoSTEM faculty voters

# of voters	**25**	**22**	**17**	**16**
1st choice	M	V	L	D

- Plurality Method – Dean Meisch wins the election

CoSTEM Dean Election

# of voters	25	20	17	8	8	2
1st choice	M	V	L	D	D	V
2nd choice	D	D	V	L	L	L
3rd choice	V	L	D	M	V	M
4th choice	L	M	M	V	M	D

- Elimination Method
  - Round 1: M – 25, V – 22, L – 17, D – 16, Dunkle is eliminated

CoSTEM Dean Election

# of voters	25	20	17	8	8	2
1st choice	M	V	L	D	D	V
2nd choice	D	D	V	L	L	L
3rd choice	V	L	D	M	V	M
4th choice	L	M	M	V	M	D

- Elimination Method
  - Round 1: M – 25, V – 22, L – 17, D – 16, Dunkle is eliminated
  - Round 2: M – 25, V – 22, L – 33, Vogel is eliminated

CoSTEM Dean Election

# of voters	25	20	17	8	8	2
1st choice	M	V	L	D	D	V
2nd choice	D	D	V	L	L	L
3rd choice	V	L	D	M	V	M
4th choice	L	M	M	V	M	D

- Elimination Method
  - Round 1: M – 25, V – 22, L – 17, D – 16, Dunkle is eliminated
  - Round 2: M – 25, V – 22, L – 33, Vogel is eliminated
  - Final Round: M – 25, L – 55, Lee is the winner

CoSTEM Dean Election

# of voters	25	20	17	8	8	2
1st choice	M	V	L	D	D	V
2nd choice	D	D	V	L	L	L
3rd choice	V	L	D	M	V	M
4th choice	L	M	M	V	M	D

- Were either of these outcomes unfair?
  - No candidate had a majority of first-place votes, so the Majority Criterion is satisfied
  - Meisch has 45 last place votes, so the Plurality Method violated the Unfavorable Majority Criterion

CoSTEM Dean Election

# of voters	25	20	17	8	8	2
1st choice	M	V	L	D	D	V
2nd choice	D	D	V	L	L	L
3rd choice	V	L	D	M	V	M
4th choice	L	M	M	V	M	D

- Was the Elimination Method fair?
  - Condorcet Criterion?

CoSTEM Dean Election

# of voters	25	20	17	8	8	2
1st choice	M	V	L	D	D	V
2nd choice	D	D	V	L	L	L
3rd choice	V	L	D	M	V	M
4th choice	L	M	M	V	M	D

- Was the Elimination Method fair?
  - Dunkle wins 53 to 27 over Meisch

CoSTEM Dean Election

# of voters	25	20	17	8	8	2
1st choice	M	V	L	D	D	V
2nd choice	D	D	V	L	L	L
3rd choice	V	L	D	M	V	M
4th choice	L	M	M	V	M	D

- Was the Elimination Method fair?
  - Dunkle wins 53 to 27 over Meisch
  - Dunkle wins 41 to 39 over Vogel

CoSTEM Dean Election

# of voters	25	20	17	8	8	2
1st choice	M	V	L	D	D	V
2nd choice	D	D	V	L	L	L
3rd choice	V	L	D	M	V	M
4th choice	L	M	M	V	M	D

- Was the Elimination Method fair?
  - Dunkle wins 53 to 27 over Meisch
  - Dunkle wins 41 to 39 over Vogel
  - Dunkle wins 61 to 19 over Lee
  - Dunkle is favored over all candidates, so the Condorcet Criterion was violated by BOTH methods

CoSTEM Dean Election

# of voters	25	20	17	8	8	2
1st choice	M	V	L	D	D	V
2nd choice	D	D	V	L	L	L
3rd choice	V	L	D	M	V	M
4th choice	L	M	M	V	M	D

- Elimination Method Revisited – Some Vogel supporters moved Lee up to their top choice

CoSTEM Dean Election

# of voters	25	11	17	8	8	2	9
1st choice	M	V	L	D	D	V	L
2nd choice	D	D	V	L	L	L	V
3rd choice	V	L	D	M	V	M	D
4th choice	L	M	M	V	M	D	M

- Elimination Method Revisited – Some Vogel supporters moved Lee up to their top choice
  - Round 1: M – 25, V – 13, L – 26, D – 16, Vogel is eliminated

CoSTEM Dean Election

# of voters	25	11	17	8	8	2	9
1st choice	M	V	L	D	D	V	L
2nd choice	D	D	V	L	L	L	V
3rd choice	V	L	D	M	V	M	D
4th choice	L	M	M	V	M	D	M

- Elimination Method Revisited – Some Vogel supporters moved Lee up to their top choice
  - Round 1: M – 25, V – 13, L – 26, D – 16, Vogel is eliminated
  - Round 2: M – 25, L – 28, D – 27, Meisch is eliminated

CoSTEM Dean Election

# of voters	25	11	17	8	8	2	9
1st choice	M	V	L	D	D	V	L
2nd choice	D	D	V	L	L	L	V
3rd choice	V	L	D	M	V	M	D
4th choice	L	M	M	V	M	D	M

- Elimination Method Revisited – Some Vogel supporters moved Lee up to their top choice
  - Round 1: M – 25, V – 13, L – 26, D – 16, Vogel is eliminated
  - Round 2: M – 25, L – 28, D – 27, Meisch is eliminated
  - Final Round: L – 28, D – 52, Dunkle is the winner
- This violates the Monotonicity Criterion

A Better Voting Method?

- Method of Pairwise Comparisons
  - Compare all candidates in head-to-head competition, winner is the candidate that wins the most matchups
  - Would always satisfy the Majority, Unfavorable Majority, Condorcet, and Monotonicity Criteria

A Better Voting Method?

- Borda Method (points-based system)
  - Used in Top 25 polls
  - For an election with n candidates, a 1st place vote earns a candidates n points, second place $n-1$ points, …, last place 1 point
  - Candidate who earns the most points wins

FCS Coaches Poll

RANK	TEAM	TREND	POINTS
1	S. Dakota St. (26) 9-0	–	650
2	Furman 8-1	–	612
3	Montana 8-1	–	603
4	Idaho 7-2	▲1	575
5	Montana St. 7-2	▲3	533
6	South Dakota 7-2	▲6	492
7	NC Central 8-1	▲2	491
8	Delaware 7-2	▼2	386
9	Florida A&M 8-1	▲4	378
10	Incarnate Word 7-2	▼6	368
11	Sacramento St. 6-3	▼4	355
12	North Dakota 6-3	▲3	346
13	N. Dakota St. 6-3	▼3	324
14	Austin Peay 7-2	▲3	304

A Better Voting Method?

- Extended Borda Method
  - Used in Mario Kart and NASCAR
  - Points difference between 1st and 2nd > 2nd and 3rd ≥ 3rd and 4th…
  - Candidate who earns the most points wins
  - Versions are used in Iceland, Kiribati, and Nauru

Any questions?